KVS ERNAKULAM REGION – PRE-BOARD EXAMINATION

MATHEMATICS (Code-041) – CLASS XII

ANSWER KEY WITH STEP-WISE MARKS

SECTION A (1 Mark Each)

Q1. (c) 40

Q2. (a) [0, 1]

Q3. (b) 2

Q4. (a) 1/x

Q5. (a) 0

Q6. (c) 4

Q7. (a) Order 2, Degree 1

Q8. (d) 512

Q9. (b) -π/3

Q10. (b) O (Zero matrix)

Q11. (b) π/4

Q12. (b) e^(1/e)

Q13. (a) (1, -2, 3)

Q14. (b) Convex polygon

Q15. (a) log(1 + e^x) + C

Q16. (b) (-∞, -1) ∪ (1, ∞)

Q17. (a) 0

Q18. (b) open half plane not containing origin.

Q19. (c) (A) is true but (R) is false.

Q20. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

SECTION B (2 Marks Each)

Q21. $\sin^{-1}(\cos(3\pi/5)) = \sin^{-1}(\sin(\pi/2 - 3\pi/5))$ **[1 mark]**
$= \sin^{-1}(\sin(-\pi/10)) = -\pi/10$ **[1 mark]**.

Q22. $dx/d\theta = -3a\cos^2\theta\sin\theta$, $dy/d\theta = 3a\sin^2\theta\cos\theta$ **[1 mark]**
$dy/dx = (dy/d\theta)/(dx/d\theta) = -\tan\theta$
At $\theta = \pi/4$: $dy/dx = -\tan(\pi/4) = -1$ **[1 mark]**.

Q23. $f(x) = 2x^3 - 9x^2 + 12x + 15 \Rightarrow f'(x) = 6x^2 - 18x + 12 = 6(x - 1)(x - 2)$ **[1 mark]**
For strictly decreasing, $f'(x) < 0 \Rightarrow x \in (1, 2)$ **[1 mark]**.
[OR]
$f'(x) = 3x^2 - 3 = 0 \Rightarrow x = 1 \in [0, 2]$ **[1 mark]**.
$f(0) = 2$, $f(1) = 0$, $f(2) = 4$.
Absolute Max = 4 (at x = 2), Absolute Min = 0 (at x = 1) **[1 mark]**.

Q24. $\mathbf{a} \times \mathbf{b} = |i\ j\ k;\ 1\ 1\ 1;\ 1\ 2\ 3| = i(3 - 2) - j(3 - 1) + k(2 - 1) = i - 2j + k$ **[1 mark]**.
$|\mathbf{a} \times \mathbf{b}| = \sqrt{(1^2 + (-2)^2 + 1^2)} = \sqrt{6}$.
Unit vector $= \pm(i - 2j + k)/\sqrt{6}$ **[1 mark]**.

Q25. $dy/dx = \cos(x + y) \cdot (1 + dy/dx)$. Given $dy/dx = 0$ **[1 mark]**.
$\Rightarrow 0 = \cos(x + y)(1 + 0) \Rightarrow \cos(x + y) = 0$.
Using $\sin^2(x + y) + \cos^2(x + y) = 1 \Rightarrow y^2 + 0 = 1 \Rightarrow y = \pm 1$ **[1 mark]**.

SECTION C (3 Marks Each)

Q26. Let $x^2 = t$: $t/[(t + 1)(t + 4)] = -1/[3(t + 1)] + 4/[3(t + 4)]$ **[1 mark]**.
Integrand $= -1/[3(x^2 + 1)] + 4/[3(x^2 + 4)]$ **[1 mark]**.
$\int = -(1/3)\tan^{-1}(x) + (4/3)\cdot(1/2)\tan^{-1}(x/2) + C$
$= -(1/3)\tan^{-1}(x) + (2/3)\tan^{-1}(x/2) + C$ **[1 mark]**.

Q27. Let $I = \int_0^{\pi} (x\sin x)/(1 + \cos^2 x)\,dx$. Using property $\int_0^a f(x) = \int_0^a f(a-x)$:
$2I = \pi \int_0^{\pi} (\sin x)/(1 + \cos^2 x)\,dx$ **[1 mark]**.
Put $\cos x = t$, $-\sin x\,dx = dt$:
$2I = \pi \int_{-1}^{1} dt/(1 + t^2) = \pi\,[\tan^{-1}(t)]_{-1}^{1}$ **[1 mark]**.
$2I = \pi[\pi/4 - (-\pi/4)] = \pi^2/2 \Rightarrow I = \pi^2/4$ **[1 mark]**.
[OR]
$\int 1/\sqrt{(7 - 6x - x^2)}\,dx = \int 1/\sqrt{[16 - (x + 3)^2]}\,dx$ **[1.5 marks]**
$= \sin^{-1}((x + 3)/4) + C$ **[1.5 marks]**.

Q28. Linear D.E. $dy/dx + P\,y = Q$, where $P = \cot x$.
I.F. $= e^{\int \cot x\,dx} = e^{\ln \sin x} = \sin x$ **[1 mark]**.
$y \cdot \sin x = \int (2x\sin x + x^2\cos x)\,dx$ **[1 mark]**.
$y \cdot \sin x = \int d(x^2 \sin x) = x^2 \sin x + C \Rightarrow y = x^2 + C\,\mathrm{cosec}\,x$ **[1 mark]**.

Q29. $dy/dx = (y^2 - x^2)/(2xy)$. Homogeneous D.E. Put $y = vx$ **[1 mark]**.
$v + x(dv/dx) = (v^2 - 1)/(2v) \Rightarrow x(dv/dx) = -(v^2 + 1)/(2v)$
$\int 2v/(v^2 + 1)\,dv = -\int dx/x$ **[1 mark]**.
$\ln(v^2 + 1) = -\ln|x| + \ln C \Rightarrow (v^2 + 1)x = C \Rightarrow x^2 + y^2 = Cx$ **[1 mark]**.

Q30. $\mathbf{a}_1 = i + j$, $\mathbf{b}_1 = i - j + k$; $\mathbf{a}_2 = 2i + j - k$, $\mathbf{b}_2 = 2i + j + 2k$.
$\mathbf{a}_2 - \mathbf{a}_1 = i - k$. $\mathbf{b}_1 \times \mathbf{b}_2 = |i\ j\ k;\ 1\ -1\ 1;\ 2\ 1\ 2| = -3i + 3k$ **[1.5 marks]**.

$d = |(a_2 - a_1) \cdot (b_1 \times b_2)| / |b_1 \times b_2| = |-3 - 3| / \sqrt{(9 + 9)} = 6 / 3\sqrt{2}$ **[1 mark]**.
Shortest Distance = $\sqrt{2}$ units **[0.5 mark]**.
[OR]
Direction vectors: $b_1 = (3, -16, 7)$; $b_2 = (-3, 8, -5)$ **[1 mark]**.
Vector parallel to required line = $b_1 \times b_2 = 24i - 6j - 24k = 6(4i - j - 4k)$ **[1 mark]**.
Required vector equation: $r = (i + 2j - 4k) + \lambda(4i - j - 4k)$ **[1 mark]**.

Q31. Constraints: $x + y \le 50$, $3x + y \le 90$, $x \ge 0$, $y \ge 0$.
Correct Graph and shading of feasible region **[1 mark]**.
Corner points: (0, 0), (30, 0), (20, 30), (0, 50) **[1 mark]**.
Z values: Z(0,0)=0, Z(30,0)=120, Z(20,30)=110, Z(0,50)=50.
Maximum value of Z = 120 at (30, 0) **[1 mark]**.

SECTION D (5 Marks Each)

Q32. $|A| = 1(6 - 4) - (-1)(4 - 0) + 0 = 6 \neq 0$ **[1 mark]**.
Co-factors: $A_{11}=2$, $A_{12}=-4$, $A_{13}=2$; $A_{21}=2$, $A_{22}=2$, $A_{23}=-1$; $A_{31}=-4$, $A_{32}=-4$, $A_{33}=5$ **[1.5 marks]**.
adj A = [[2, 2, -4], [-4, 2, -4], [2, -1, 5]]. $A^{-1} = (1/6)$ adj A **[0.5 mark]**.
$X = A^{-1}B = (1/6)$ [[2, 2, -4], [-4, 2, -4], [2, -1, 5]] $[3, 17, 7]^T$ **[1 mark]**.
$X = (1/6)[12, -6, 24]^T = [2, -1, 4]^T \Rightarrow x = 2, y = -1, z = 4$ **[1 mark]**.

Q33. Ellipse: $x^2/9 + y^2/4 = 1 \Rightarrow y = (2/3)\sqrt{(9 - x^2)}$. (Figure / Setup) **[1 mark]**.
Total Area = $4 \times \int_0^3 (2/3)\sqrt{(9 - x^2)}\, dx$ **[1 mark]**.
Formula: $(8/3)\ [\ (x/2)\sqrt{(9 - x^2)} + (9/2) \sin^{-1}(x/3)\ ]_0^3$ **[1.5 marks]**.
$= (8/3)\ [\ (9/2)(\pi/2)\ ] = 6\pi$ sq. units **[1.5 marks]**.
[OR]
Intersection of $y^2 = 4x$ and $x^2 + y^2 = 9/4$: $x^2 + 4x - 9/4 = 0 \Rightarrow x = 1/2$ **[1 mark]**.
Area = 2 [∫₀^(1/2) 2√x dx + ∫_(1/2)^(3/2) √(9/4 - x²) dx] **[1.5 marks]**.
Evaluation of first integral: = 4(2/3)(1/2)^(3/2) = √2/3 **[1 mark]**.
Evaluation of second integral and sum: = √2/6 + (9π/8) - (9/4) sin⁻¹(1/3) sq. units **[1.5 marks]**.

Q34. Relation R on N × N: (a, b) R (c, d) ⇔ a + d = b + c.
1. Reflexive: $a + b = b + a \Rightarrow$ (a, b) R (a, b). R is reflexive **[1.5 marks]**.
2. Symmetric: (a, b) R (c, d) ⇒ a + d = b + c ⇒ c + b = d + a ⇒ (c, d) R (a, b). R is symmetric **[1.5 marks]**.
3. Transitive: Let (a, b) R (c, d) and (c, d) R (e, f).
a + d = b + c and c + f = d + e **[1 mark]**.
Adding: a + d + c + f = b + c + d + e ⇒ a + f = b + e ⇒ (a, b) R (e, f). Transitive **[1 mark]**.

Q35. General point Q on line: Q(3λ + 6, 2λ + 7, -2λ + 7) **[1 mark]**.
Direction ratios of PQ: (3λ + 5, 2λ + 5, -2λ + 4) **[1 mark]**.
PQ ⊥ line: $3(3\lambda + 5) + 2(2\lambda + 5) - 2(-2\lambda + 4) = 0 \Rightarrow 17\lambda + 17 = 0 \Rightarrow \lambda = -1$ **[1.5 marks]**.
Foot of perpendicular Q: (3, 5, 9) **[0.5 mark]**.
Length of perpendicular = $\sqrt{((3 - 1)^2 + (5 - 2)^2 + (9 - 3)^2)} = \sqrt{49} = 7$ units **[1 mark]**.

SECTION E (4 Marks Each)

Q36. (i) System: $2x + y + 3z = 23000$; $x + 2y + 2z = 21000$; $3x + y + z = 18000$ **[1 mark]**.
(ii) Matrix form: $[[2, 1, 3], [1, 2, 2], [3, 1, 1]]\ [x, y, z]^T = [23000, 21000, 18000]^T$ **[1 mark]**.
(iii) $|A| = -10 \neq 0$. Since $|A| \neq 0$, A is non-singular and invertible **[1 mark]**.
(iv) Solving $X = A^{-1}B$ gives: x = ₹3,000, y = ₹5,000, z = ₹4,000 **[1 mark]**.

Q37. Given height equation: $y = 4x - (1/2)x^2$ (or $y = 4x - x^2$)
a. Rate of growth: $dy/dx = 4 - x$ (or $4 - 2x$) **[1 mark]**.
b. Days for maximum height: Set $dy/dx = 0 \Rightarrow x = 4$ days (or 2 days) **[1 mark]**.
c. Maximum height: $y_{max} = 4(4) - (1/2)(4)^2 = 8$ cm (or 4 cm) **[1 mark]**.
d. Height after 2 months (60 days): Growth stops once max height is attained; remains 8 cm (or 4 cm) **[1 mark]**.

Q38. Given profit function: $P(x) = -5x^2 + 125x + 37500$
1. Production for max profit: $P'(x) = -10x + 125 = 0 \Rightarrow x = 12.5$ units **[1 mark]**.
2. Maximum profit: $P(12.5) = -5(12.5)^2 + 125(12.5) + 37500$ = ₹38,281.25 **[1 mark]**.
3. Strictly increasing interval: $P'(x) > 0 \Rightarrow -10x + 125 > 0 \Rightarrow x \in$ **[0, 12.5) [1 mark]**.
4. Profit at 2 units: $P(2) = -5(2)^2 + 125(2) + 37500$ = ₹37,730 **[1 mark]**.