MARKING SCHEME

MATHEMATICS (041)

Sample Paper 01 • Class XII • Session 2026–27

Q.P. Code : MC/SP/01 **Maximum Marks : 80**

Step-by-step solutions with the value points and marks for each step, as per the CBSE marking scheme. Marks for each step are shown in the right-hand column.

SECTION A

1. **Correct answer: (C) $-\frac{2\pi}{5}$** **1**

The principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, and $\frac{3\pi}{5}$ does not lie in it.

$\tan\frac{3\pi}{5} = \tan\left(\frac{3\pi}{5} - \pi\right) = \tan\left(-\frac{2\pi}{5}\right)$, and $-\frac{2\pi}{5} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; so the value is $-\frac{2\pi}{5}$.

2. **Correct answer: (D) [1, 2]** **1**

$\sin^{-1}$ is defined only on $[-1, 1]$, so $-1 \le 2x - 3 \le 1$.

$2 \le 2x \le 4$, i.e. $1 \le x \le 2$.

3. **Correct answer: (B) 1** **1**

$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ (principal value), so the angle is $\frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$.

$\sin\frac{\pi}{2} = 1$.

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4. **Correct answer: (A) m = 3, n = 5** **1**

Two matrices can be added or subtracted only when they have the same order.

So $3 \times n = m \times 5$, giving $m = 3$ and $n = 5$.

5. **Correct answer: (B) ±8** **1**

For a square matrix of order n, $|\text{adj } A| = |A|^{n-1}$; here $|\text{adj } A| = |A|^2$.

$|A|^2 = 64 \Rightarrow |A| = \pm 8$.

6. **Correct answer: (D) a = −2, b = 3** **1**

A is skew-symmetric when $A' = -A$, i.e. $a_{ji} = -a_{ij}$ for all i, j.

$a_{12} = -a_{21} \Rightarrow a = -2$; $a_{31} = -a_{13} \Rightarrow b = 3$.

7. **Correct answer: (A) 1** **1**

$1 - \cos 4x = 2\sin^2 2x$, so $f(x) = \frac{2\sin^2 2x}{8x^2} = \left(\frac{\sin 2x}{2x}\right)^2$ for $x \neq 0$.

The limit as $x \to 0$ is 1, so continuity needs $k = f(0) = 1$.

8. **Correct answer: (C) 2** **1**

Put $x = \cos\theta$, $\theta \in \left(0, \frac{\pi}{2}\right)$: $2x^2 - 1 = \cos 2\theta$ with $2\theta \in (0, \pi)$, so $\cos^{-1}(2x^2 - 1) = 2\theta = 2\cos^{-1}x$.

Its derivative with respect to $\cos^{-1}x$ is therefore 2.

9. **Correct answer: (D) $(-\infty, 1)$** **1**

$f'(x) = e^{-x} - xe^{-x} = e^{-x}(1 - x)$.

Since $e^{-x} > 0$, $f'(x) > 0$ exactly when $x < 1$.

10. **Correct answer: (B) 8** **1**

$f'(x) = 4 - x = 0 \Rightarrow x = 4$ (the only critical point).

$f(-2) = -10$, $f(4) = 8$, $f\left(\frac{9}{2}\right) = \frac{63}{8}$; the greatest is 8.

11. **Correct answer: (C) xy = C** **1**

$x\,dy + y\,dx = d(xy)$, so the equation says $d(xy) = 0$.

Integrating: $xy = C$.

12. **Correct answer: (A) $\log|\log x| + C$** **1**

Put $t = \log x$, $dt = \frac{dx}{x}$.

$\int \frac{dt}{t} = \log|t| + C = \log|\log x| + C$.

13. **Correct answer: (C) 3** **1**

$(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = |\vec{x}|^2 - |\vec{a}|^2 = |\vec{x}|^2 - 1$.

$|\vec{x}|^2 = 9 \Rightarrow |\vec{x}| = 3$.

14. **Correct answer: (B) $\frac{\sqrt{62}}{2}$** **1**

Area $= \frac{1}{2}|d_1 \times d_2|$, and $d_1 \times d_2 = -2\hat{i} + 3\hat{j} + 7\hat{k}$.

$|d_1 \times d_2| = \sqrt{4+9+49} = \sqrt{62}$, so the area is $\frac{\sqrt{62}}{2}$.

15. **Correct answer: (A) $\frac{\pi}{6}$** **1**

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta = 3 \times \frac{2}{3} \sin\theta = 2 \sin\theta$.

$2 \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{6}$.

16. **Correct answer: (D) $\frac{2}{3}$** **1**

$P(A \cap B) = P(A)\,P(B|A) = 0.4 \times 0.5 = 0.2$.

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.3} = \frac{2}{3}$.

17. **Correct answer: (B) $\frac{1}{221}$** **1**

P(first king) $= \frac{4}{52}$; given that, P(second king) $= \frac{3}{51}$ (multiplication theorem).

$P = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$.

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18. **Correct answer: (A)** $\frac{2}{3}$ **1**

The problem is not solved only if both fail: P = $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right) = \frac{1}{3}$ (independence).

P(solved) = $1 - \frac{1}{3} = \frac{2}{3}$.

19. **Correct answer: (A)** **1**

R is a standard property of definite integrals — R is true.

f(x) = x|x| satisfies f(−x) = −x|x| = −f(x), so it is odd and the integral is 0 by R. A is true and R explains it.

20. **Correct answer: (D)** **1**

R is the definition of degree — R is true.

The equation contains sin(dy/dx), so it is not a polynomial in its derivatives and its degree is not defined. A is false.

SECTION B

21. **Solution:**

1 + sin 2x = sin²x + cos²x + 2 sin x cos x = (sin x + cos x)², and sin x + cos x > 0 on $\left[0, \frac{\pi}{4}\right]$. **1**

$\int_0^{\frac{\pi}{4}} (\sin x + \cos x)\,dx$ = [−cos x + sin x] from 0 to $\frac{\pi}{4} = \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (-1 + 0) = 1$. **1**

∴ **1**

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OR

Put t = tan x, dt = sec²x dx: $\int \frac{dt}{\sqrt{t^2+4}}$. **1**

Using $\int \frac{dt}{\sqrt{t^2+a^2}} = \log|t + \sqrt{t^2+a^2}| + C$: the answer is $\log|\tan x + \sqrt{\tan^2 x + 4}| + C$. **1**

∴ **$\log|\tan x + \sqrt{(\tan^2 x + 4)}| + C$**

22. **Solution:**

Chain rule: $\frac{dy}{dx} = \frac{1}{\sqrt{1-(x\sqrt{x})^2}} \times \frac{d}{dx}$(x^(3/2)) = $\frac{1}{\sqrt{1-x^3}} \times \frac{3}{2}\sqrt{x}$. **1**

$\frac{dy}{dx} = \frac{3\sqrt{x}}{2\sqrt{1-x^3}}$. **1**

∴ $\frac{3\sqrt{x}}{2\sqrt{1-x^3}}$

23. **Solution:**

(A) Three Dimensional Geometry

Divide each by 6: 2x = 3y = −z gives $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ (direction ratios 3, 2, −6); 6x = −y = −4z gives $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$ (direction ratios 2, −12, −3). **1**

$a_1a_2 + b_1b_2 + c_1c_2 = 6 - 24 + 18 = 0$, so cos θ = 0 and the lines are perpendicular: θ = 90°. **1**

∴ **90°**

OR

(B) Vector Algebra

$\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{a} - \vec{b} = -\hat{j} - 2\hat{k}$; their cross product = $-2\hat{i} + 4\hat{j} - 2\hat{k}$, with magnitude $\sqrt{24} = 2\sqrt{6}$. **1**

Required unit vector = $\pm\frac{1}{\sqrt{6}}(-\hat{i} + 2\hat{j} - \hat{k})$. **1**

∴ **$\pm(1/\sqrt{6})(-\hat{i} + 2\hat{j} - \hat{k})$**

24. Solution:

$\vec{a} \cdot (\vec{b} + \vec{c}) = 0$, $\vec{b} \cdot (\vec{c} + \vec{a}) = 0$, $\vec{c} \cdot (\vec{a} + \vec{b}) = 0$; adding, $2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$. **1**

$|\vec{a} + \vec{b} + \vec{c}|^2 = 9 + 16 + 25 + 0 = 50 \Rightarrow |\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$. **1**

∴ **$5\sqrt{2}$**

25. Solution:

Z = x + 2y at the corner points: (0, 10): 20; (5, 5): 15; (15, 15): 45; (0, 20): 40. **1**

For a bounded region the extreme values occur at corner points: maximum Z = 45 at (15, 15), minimum Z = 15 at (5, 5). **1**

∴ **Max 45 at (15, 15); Min 15 at (5, 5)**

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SECTION C

26. Solution:

Quantity matrix Q = $\begin{bmatrix} 20 & 30 & 10 \\ 25 & 20 & 15 \\ 30 & 25 & 20 \end{bmatrix}$ (rows: sections I, II, III) and price matrix P = $\begin{bmatrix} 40 \\ 10 \\ 300 \end{bmatrix}$. **1**

QP = $\begin{bmatrix} 800+300+3000 \\ 1000+200+4500 \\ 1200+250+6000 \end{bmatrix} = \begin{bmatrix} 4100 \\ 5700 \\ 7450 \end{bmatrix}$. **1**

Section I: ₹4100, Section II: ₹5700, Section III: ₹7450; total = ₹17,250. **1**

∴ **₹4100, ₹5700, ₹7450; total ₹17,250**

27. Solution:

f′(x) = cos x − sin x = 0 ⇒ tan x = 1 ⇒ x = $\frac{\pi}{4}, \frac{5\pi}{4}$ in [0, 2π]. **1**

On $\left[0, \frac{\pi}{4}\right) \wedge \left(\frac{5\pi}{4}, 2\pi\right]$, cos x > sin x, so f′(x) > 0: f is strictly increasing there. **1**

On $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$, cos x < sin x, so f′(x) < 0: f is strictly decreasing there. **1**

∴ **Increasing on [0, π/4) ∪ (5π/4, 2π]; decreasing on (π/4, 5π/4)**

28. Solution:

$\frac{dy}{dx} = 2\tan^{-1}x \times \frac{1}{1+x^2} \Rightarrow (1 + x^2)\frac{dy}{dx} = 2\tan^{-1}x.$ **1**

Differentiating again: $(1 + x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \frac{2}{1+x^2}.$ **1**

Multiplying by $(1 + x^2)$: $(x^2 + 1)^2\frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} = 2$. Hence proved. **1**

∴ **Proved**

OR

$\frac{dx}{dt} = a(-\sin t + \sin t + t\cos t) = at\cos t$; $\frac{dy}{dt} = a(\cos t - \cos t + t\sin t) = at\sin t.$ **1**

$\frac{dy}{dx} = \frac{at\sin t}{at\cos t} = \tan t.$ **1**

$\frac{d^2y}{dx^2} = \sec^2 t \times \frac{dt}{dx} = \frac{sec^2 t}{at\cos t} = \frac{sec^3 t}{at}.$ **1**

∴ **$\sec^3 t$ / (at)**

29. Solution:

Rewrite as $\frac{dx}{dy} = \frac{x+2y^2}{y}$, i.e. $\frac{dx}{dy} - \frac{1}{y}x = 2y$, a linear equation in x. **1**

Integrating factor = $e^{\wedge}\left(-\int\frac{dy}{y}\right) = \frac{1}{y}$; so $\frac{x}{y} = \int 2y \times \frac{1}{y}\,dy = 2y + C.$ **1**

General solution: $x = 2y^2 + Cy.$ **1**

∴ **$x = 2y^2 + Cy$**

OR

$\frac{dy}{dx} = \frac{x+y}{x-y}$ is homogeneous; put $y = vx$: $v + x\frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{1-v}.$ **1**

$\frac{1-v}{1+v^2}dv = \frac{dx}{x} \Rightarrow \tan^{-1}v - \frac{1}{2}\log(1 + v^2) = \log|x| + C.$ **1**

With $v = \frac{y}{x}$: $\tan^{-1}\frac{y}{x} = \frac{1}{2}\log(x^2 + y^2) + C.$ **1**

∴ **$\tan^{-1}(y/x) = ½\log(x^2 + y^2) + C$**

30. Solution:

A line perpendicular to both is parallel to $(3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k}) = 24\hat{i} + 36\hat{j} + 72\hat{k} = 12(2\hat{i} + 3\hat{j} + 6\hat{k})$. **1**

Vector equation: $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}).$ **1**

Cartesian equation: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}.$ **1**

∴ **$(x - 1)/2 = (y - 2)/3 = (z + 4)/6$**

OR

Line BC: any point P = (2t, −1 − 2t, 3 − 4t), with direction (2, −2, −4). **1**

$\overrightarrow{AP}$ = (2t + 1, −9 − 2t, −1 − 4t) is perpendicular to BC: 2(2t + 1) − 2(−9 − 2t) − 4(−1 − 4t) = 24t + 24 = 0 ⇒ t = −1. **1**

Foot of the perpendicular = (−2, 1, 7). **1**

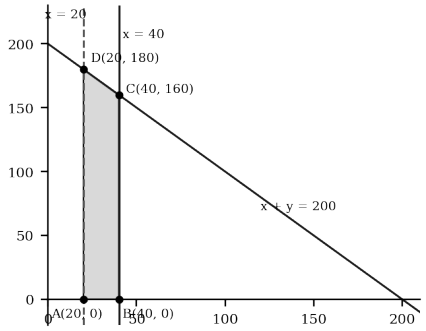
∴ **(−2, 1, 7)**

31. Solution:

Draw x + y = 200, x = 40 and x = 20. The feasible region is the bounded region ABCD shown, with corners A(20, 0), B(40, 0), C(40, 160), D(20, 180). **1**

Z at the corners: A: 8000; B: 16000; C: 16000 + 48000 = 64000; D: 8000 + 54000 = 62000. **1**

Maximum Z = 64,000 at x = 40, y = 160. **1**



∴ **Max Z = 64,000 at (40, 160)**

SECTION D

32. Solution:

Reflexive: a + b = b + a for every (a, b), so (a, b) R (a, b). **1**

Symmetric: if (a, b) R (c, d), then a + d = b + c, i.e. c + b = d + a, so (c, d) R (a, b). **1**

Transitive: if a + d = b + c and c + f = d + e, adding gives a + d + c + f = b + c + d + e, i.e. a + f = b + e, so (a, b) R (e, f). **1**

Hence R is reflexive, symmetric and transitive: an equivalence relation. **1**

(a, b) R (2, 5) ⇔ a + 5 = b + 2 ⇔ b = a + 3. Equivalence class of (2, 5) = {(1, 4), (2, 5), (3, 6), (4, 7), …} = {(a, a + 3) : a ∈ N}. **1**

∴ **Equivalence relation; [(2, 5)] = {(a, a + 3) : a ∈ N}**

OR

Not one-one: f(1) = (1 + 1)/2 = 1 and f(2) = 2/2 = 1, but 1 ≠ 2. **1**

Onto: take any m ∈ N. The even number 2m ∈ N has f(2m) = m. **1**

Also 2m − 1 (odd) gives f(2m − 1) = m, so every m has at least one pre-image. **1**

Therefore the range of f is all of N, and f is onto. **1**

Hence f is onto but not one-one. **1**

∴ **Onto, not one-one**

33. Solution:

Let I = $\int_0^{\frac{\pi}{2}} \log(\sin x)\,dx$. Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$: I = $\int_0^{\frac{\pi}{2}} \log(\cos x)\,dx$. **1**

Adding: $2I = \int_0^{\frac{\pi}{2}} \log(\sin x \cos x)\,dx = \int_0^{\frac{\pi}{2}} \log(\sin 2x)\,dx - \frac{\pi}{2}\log 2.$ **1**

In $\int_0^{\frac{\pi}{2}} \log(\sin 2x)\,dx$ put 2x = t: it becomes $\frac{1}{2}\int_0^{\pi} \log(\sin t)\,dt = \frac{1}{2} \times 2\int_0^{\frac{\pi}{2}} \log(\sin t)\,dt = I$ **1**

$\left(using \int_0^{2a} f = 2\int_0^{a} f \; when \; f(2a-x) = f(x)\right).$

So $2I = I - \frac{\pi}{2}\log 2.$ **1**

$I = -\frac{\pi}{2}\log 2.$ **1**

∴ **−(π/2) log 2**

34. Solution:

In the first quadrant the circle is $y = \sqrt{(16 - x^2)}$; the required region lies under this arc, above the x-axis, from x = 0 to x = 2 (see figure). **1**

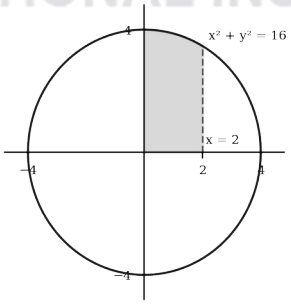
Area = $\int_0^2 \sqrt{16 - x^2}\,dx.$ **1**

Using the standard integral $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$ with a = 4. **1**

Area = $\left[\frac{x}{2}\sqrt{16 - x^2} + 8\sin^{-1}\frac{x}{4}\right]$ from 0 to 2 = $\left(1 \times \sqrt{12} + 8 \times \frac{\pi}{6}\right) - 0.$ **1**

= $\left(2\sqrt{3} + \frac{4\pi}{3}\right)$ square units. **1**

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∴ **(2√3 + 4π/3) sq. units**

OR

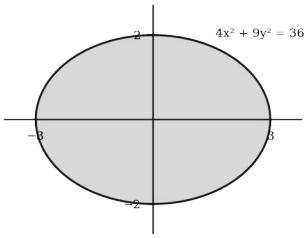
The ellipse is $\frac{x^2}{9} + \frac{y^2}{4} = 1$ (a = 3, b = 2); in the first quadrant $y = \frac{2}{3}\sqrt{9 - x^2}.$ **1**

By symmetry about both axes, Area = 4 × (area in the first quadrant). **1**

Area = $4\int_0^3 \frac{2}{3}\sqrt{9 - x^2}\,dx = \frac{8}{3}\int_0^3 \sqrt{9 - x^2}\,dx.$ **1**

= $\frac{8}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]$ from 0 to 3 = $\frac{8}{3} \times \frac{9}{2} \times \frac{\pi}{2}.$ **1**

= 6π square units. **1**



∴ **6π sq. units**

35. Solution:

Let E_1: hosteller, E_2: day scholar, A: gets an A grade. P(E_1) = 0.6, P(E_2) = 0.4, P(A|E_1) = 0.3, P(A|E_2) = 0.2. **1**

(a) Theorem of total probability: P(A) = 0.6 × 0.3 + 0.4 × 0.2 = 0.18 + 0.08 = 0.26. **1**

(b) Bayes' theorem: P(E_1|A) = $\frac{0.6 \times 0.3}{0.26} = \frac{0.18}{0.26}$. **1**

= $\frac{9}{13}$. **1**

(c) P(E_2|A) = $1 - \frac{9}{13} = \frac{4}{13}$. **1**

∴ **(a) 0.26 (b) 9/13 (c) 4/13**

SECTION E

36. Solution:

(i)

$\vec{r} = (2\hat{i} - \hat{j}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$. **1**

(ii)

|3î + 2ĵ + 6k̂| = {sqrt|9 + 4 + 36} = 7, so the direction cosines are {frac|3|7}, {frac|2|7}, {frac|6|7}. **1**

(iii)(a)

Any point on the path: P(2 + 3λ, −1 + 2λ, 6λ); TP = (3λ − 5, 2λ + 1, 6λ − 6) must be perpendicular to the path. **1**

3(3λ − 5) + 2(2λ + 1) + 6(6λ − 6) = 49λ − 49 = 0 ⇒ λ = 1; nearest point (5, 1, 6). **1**

∴ **(5, 1, 6)**

OR

(iii)(b)

The foot of the perpendicular from T to the path is F(5, 1, 6) (λ = 1, found by making TF perpendicular to the path). **1**

TF = $\sqrt{(7-5)^2+(-2-1)^2+(6-6)^2} = \sqrt{13}$ units. **1**

∴ **√13 units**

37. Solution:

(i)

The wire covers two sides of length x and one side of length y: 2x + y = 200. **1**

(ii)

$A = xy = x(200 - 2x) = 200x - 2x^2$, $0 < x < 100$. **1**

(iii)(a)

$A'(x) = 200 - 4x = 0 \Rightarrow x = 50$; $A''(x) = -4 < 0$, so A is maximum at x = 50. **1**

Then y = 100: the garden is 50 m × 100 m, and the maximum area = 50 × 100 = 5000 m². **1**

∴ **50 m × 100 m; 5000 m²**

OR

(iii)(b)

$A'(x) = 200 - 4x = 0$ gives x = 50, and $A''(x) = -4 < 0$, so the area is maximum at x = 50. **1**

Then y = 200 − 100 = 100 = 2 × 50, i.e. y = 2x. Hence shown. **1**

∴ **y = 2x**

38. Solution:

(i)

$x + y + z = 6000$, $x + 3z = 11000$ and $x + z = 2y$, i.e. $x - 2y + z = 0$. So $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$. **1**

$|A| = 1(0 + 6) - 1(1 - 3) + 1(-2 - 0) = 6 + 2 - 2 = 6$. **1**

∴ **|A| = 6**

(ii)

$|A| \neq 0$, so $X = A^{-1}B = (1/|A|)(\text{adj } A)B = \frac{1}{6}\begin{bmatrix} 36000 - 33000 + 0 \\ 12000 + 0 + 0 \\ -12000 + 33000 + 0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix}$. **1**

x = 500, y = 2000, z = 3500: Honesty ₹500, Regularity ₹2000, Hard work ₹3500. **1**

∴ **₹500, ₹2000, ₹3500**

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