MARKING SCHEME

MATHEMATICS (041)

Sample Paper 02 • Class XII • Session 2026–27

Q.P. Code : MC/SP/02 **Maximum Marks : 80**

Step-by-step solutions with the value points and marks for each step, as per the CBSE marking scheme. Marks for each step are shown in the right-hand column.

SECTION A

1. **Correct answer: (D) $\frac{5\pi}{6}$** **1**

The principal value branch of $\cos^{-1}$ is $[0, \pi]$, and $\frac{7\pi}{6}$ is not in it.

$\cos\frac{7\pi}{6} = \cos\left(2\pi - \frac{7\pi}{6}\right) = \cos\frac{5\pi}{6}$, and $\frac{5\pi}{6} \in [0, \pi]$; so the value is $\frac{5\pi}{6}$.

2. **Correct answer: (A) (0, π)** **1**

cot is one-one on $(0, \pi)$ and takes every real value there.

So $\cot^{-1} : \mathbb{R} \to (0, \pi)$; the end points 0 and π are excluded because cot is not defined there.

3. **Correct answer: (C) $\frac{3\pi}{4}$** **1**

Principal values: $\tan^{-1}(1) = \frac{\pi}{4}$, $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ and $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$.

Sum $= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi + 8\pi - 2\pi}{12} = \frac{3\pi}{4}$.

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4. **Correct answer: (B) (2, 4) or (4, 2)** **1**

Equal matrices have equal corresponding entries: $x + y = 6$ and $xy = 8$.

x and y are the roots of $t^2 - 6t + 8 = 0$, i.e. 2 and 4; so $(x, y) = (2, 4)$ or $(4, 2)$.

5. **Correct answer: (C) ±6** **1**

LHS $= 2x^2 - 40$ and RHS $= 18 + 14 = 32$.

$2x^2 = 72 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$.

6. **Correct answer: (A) O** **1**

$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$, $5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$.

$A^2 - 5A + 7I = \begin{bmatrix} 8 - 15 + 7 & 0 \\ 0 & 3 - 10 + 7 \end{bmatrix} = O$, the zero matrix.

7. **Correct answer: (D) $\frac{3}{4}$** **1**

Left-hand limit $= f(2) = 4k$; right-hand limit $= 3$.

Continuity: $4k = 3 \Rightarrow k = \frac{3}{4}$.

8. **Correct answer: (B) sec x** **1**

$\frac{dy}{dx} = \frac{1}{sec\,x + \tan x} \times (\sec x \tan x + \sec^2 x)$ (chain rule).

$= \frac{sec\,x(\tan x + sec\,x)}{sec\,x + \tan x} = \sec x.$

9. **Correct answer: (D) is always increasing** **1**

$f'(x) = \sec^2 x - 1 = \tan^2 x \geq 0$, with equality only at $x = 0$.

So f is increasing on the whole interval.

10. **Correct answer: (C) e^$\left(\frac{1}{e}\right)$** **1**

Let $y = \left(\frac{1}{x}\right)^x$; $\log y = -x \log x$, and $\frac{d}{dx}(-x \log x) = -(1 + \log x) = 0 \Rightarrow x = \frac{1}{e}$ (a maximum, since the derivative changes from + to −).

Maximum value = e^$\left(\frac{1}{e}\right)$.

11. **Correct answer: (B) sec x** **1**

IF = e^(∫P dx) with P = tan x.

∫tan x dx = log sec x, so IF = e^(log sec x) = sec x.

12. **Correct answer: (A) $e^x \sin x + C$** **1**

The integrand is $e^x[f(x) + f'(x)]$ with $f(x) = \sin x$.

$\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C = e^x \sin x + C$.

13. **Correct answer: (B)** $\frac{5}{\sqrt{6}}$ **1**

Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$.

$\vec{a} \cdot \vec{b} = 2 + 2 + 1 = 5$ and $|\vec{b}| = \sqrt{6}$, so the projection is $\frac{5}{\sqrt{6}}$.

14. **Correct answer: (C)** $3\sqrt{3}$ **1**

$\cos\theta = \frac{3}{2 \times 3} = \frac{1}{2}$, so $\theta = \frac{\pi}{3}$ and $\sin\theta = \frac{\sqrt{3}}{2}$.

$|\vec{a} \times \vec{b}| = 2 \times 3 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$.

15. **Correct answer: (A) 9** **1**

Collinear vectors have proportional components.

$\frac{6}{2} = \frac{\lambda}{3} = \frac{-18}{-6} = 3 \Rightarrow \lambda = 9$.

16. **Correct answer: (D) 0.6** **1**

For independent events, $P(A \cap B) = P(A)\,P(B) = 0.2 \times 0.5 = 0.1$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.5 - 0.1 = 0.6$.

17. **Correct answer: (A)** $\frac{1}{3}$ **1**

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Sum 7 occurs in 6 equally likely outcomes: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1).

Of these, (2, 5) and (5, 2) show a 2, so P = $\frac{2}{6} = \frac{1}{3}$.

18. **Correct answer: (D) $\frac{1}{3}$** **1**

P(A ∪ B) = $\frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$.

P(A′ ∩ B′) = P((A ∪ B)′) = $1 - \frac{2}{3} = \frac{1}{3}$ (De Morgan's law).

19. **Correct answer: (A)** **1**

R is a standard property of definite integrals — R is true.

$\sin^2(\pi - x) = \sin^2 x$, so by R the integral = $2\int_0^{\frac{\pi}{2}} \sin^2 x\,dx = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$. A is true and R explains it.

20. **Correct answer: (C)** **1**

$\frac{dy}{y} = dx \Rightarrow \log y = x + C \Rightarrow y = Ae^x$; y(0) = 1 gives A = 1, so A is true.

R is false: a general solution contains arbitrary constants (as many as the order); only a particular solution is free of them.

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SECTION B

21. **Solution:**

Let I be the integral. Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, sin x and cos x interchange: I = $\int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \cos x \sin x}\,dx = -I$. **1**

2I = 0, so I = 0. **1**

∴ **0**

OR

cos 2x = (cos x + sin x)(cos x − sin x), so the integrand is $\frac{\cos x - \sin x}{\cos x + \sin x}$. **1**

The numerator is the derivative of the denominator: the integral is log|cos x + sin x| + C. **1**

∴ **log|cos x + sin x| + C**

22. **Solution:**

$\frac{dx}{d\theta} = a(1 - \cos\theta)$, $\frac{dy}{d\theta} = -a\sin\theta$; so $\frac{dy}{dx} = \frac{-\sin\theta}{1 - \cos\theta} = -\cot\frac{\theta}{2}$. **1**

At $\theta = \frac{\pi}{3}$: $\frac{dy}{dx} = -\cot\frac{\pi}{6} = -\sqrt{3}$. **1**

∴ $-\sqrt{3}$

23. **Solution:**

(A) Three Dimensional Geometry

Direction ratios of the first line: (4 − 2, 7 − 3, 8 − 4) = (2, 4, 4); of the second line: (1 + 1, 2 + 2, 5 − 1) = (2, 4, 4). **1**

The direction ratios are proportional (in fact equal), so the two lines are parallel; they are distinct lines, since (4, 7, 8) does not lie on the second line. **1**

∴ **The lines are parallel**

OR

(B) Vector Algebra

$\vec{a} + \lambda\vec{b} = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$. **1**

Perpendicular to $\vec{c}$: $3(2 - \lambda) + 1(2 + 2\lambda) + 0 = 8 - \lambda = 0 \Rightarrow \lambda = 8$. **1**

∴ **λ = 8**

24. Solution:

Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$. $\vec{a} \times \vec{c} = (z - y)\hat{i} + (x - z)\hat{j} + (y - x)\hat{k} = \hat{j} - \hat{k}$ gives z = y, x − z = 1, y − x = −1; and x + y + z = 3. **1**

So x = y + 1 and 3y + 1 = 3 ⇒ y = z = $\frac{2}{3}$, x = $\frac{5}{3}$: $\vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$. **1**

∴ **(5/3)î + (2/3)ĵ + (2/3)k̂**

25. Solution:

Z at the corner points: (0, 2): 10; (3, 0): 9; (6, 0): 18; (6, 8): 58; (0, 5): 25. **1**

Maximum Z = 58 at (6, 8); minimum Z = 9 at (3, 0). **1**

∴ **Max 58 at (6, 8); Min 9 at (3, 0)**

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SECTION C

26. Solution:

$A^2 = A \cdot A$: each diagonal entry = 1 + 4 + 4 = 9 and each off-diagonal entry = 2 + 2 + 4 = 8, so **1**

$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$.

$4A = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$ and $5I = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$. **1**

$A^2 - 4A - 5I = \begin{bmatrix} 9-4-5 & 0 & 0 \\ 0 & 9-4-5 & 0 \\ 0 & 0 & 9-4-5 \end{bmatrix}$ = O. Hence proved. **1**

∴ **Proved**

27. Solution:

$f'(x) = 6x^2 - 18x + 12 = 6(x - 1)(x - 2)$; $f'(x) = 0$ at x = 1, 2, dividing R into (−∞, 1), (1, 2), (2, ∞). **1**

On (−∞, 1) and (2, ∞), both factors have the same sign, so $f'(x) > 0$: f is strictly increasing. **1**

On (1, 2), $f'(x) < 0$: f is strictly decreasing. **1**

∴ **Increasing on (−∞, 1) ∪ (2, ∞); decreasing on (1, 2)**

28. Solution:

$\frac{dy}{dx}$ = mAe^(mx) + nBe^(nx) and $\frac{d^2y}{dx^2}$ = m²Ae^(mx) + n²Be^(nx). **1**

(m + n)$\frac{dy}{dx}$ = (m² + mn)Ae^(mx) + (mn + n²)Be^(nx), and mny = mnAe^(mx) + mnBe^(nx). **1**

$\frac{d^2y}{dx^2}$ − (m + n)$\frac{dy}{dx}$ + mny = (m² − m² − mn + mn)Ae^(mx) + (n² − mn − n² + mn)Be^(nx) = 0. **1**

∴ **Proved**

OR

Differentiating implicitly: 2(x² + y²)$\left(2x+2y\frac{dy}{dx}\right)$ = y + x$\frac{dy}{dx}$. **1**

4x(x² + y²) + 4y(x² + y²)$\frac{dy}{dx}$ = y + x$\frac{dy}{dx}$ ⇒ $\frac{dy}{dx}$[4y(x² + y²) − x] = y − 4x(x² + y²). **1**

$\frac{dy}{dx}=\frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$. **1**

∴ **dy/dx = [y − 4x(x² + y²)] / [4y(x² + y²) − x]**

29. Solution:

This is linear of the form $\frac{dy}{dx}$ + Py = Q with P = cot x, Q = 4x cosec x; IF = e^(∫cot x dx) = e^(log sin x) = sin x. **1**

y sin x = ∫ 4x cosec x × sin x dx = ∫ 4x dx = 2x² + C. **1**

At x = $\frac{\pi}{2}$, y = 0: 0 = $\frac{\pi^2}{2}$ + C ⇒ C = $-\frac{\pi^2}{2}$; so y sin x = 2x² − $\frac{\pi^2}{2}$. **1**

∴ **y sin x = 2x² − π²/2**

OR

$\frac{dy}{dx}=\frac{y+\sqrt{x^2+y^2}}{x}$ is homogeneous; put y = vx: v + x$\frac{dv}{dx}$ = v + $\sqrt{1+v^2}$ (x > 0). **1**

$\frac{dv}{\sqrt{1+v^2}}=\frac{dx}{x}$ ⇒ log|v + $\sqrt{1+v^2}$| = log|x| + log C. **1**

v + $\sqrt{1+v^2}$ = Cx ⇒ y + $\sqrt{x^2+y^2}$ = Cx². **1**

∴ **y + √(x² + y²) = Cx²**

30. Solution:

Here a_1 = (−1, −1, −1), b_1 = (7, −6, 1); a_2 = (3, 5, 7), b_2 = (1, −2, 1). $b_1 \times b_2$ = (−4, −6, −8), with $|b_1 \times b_2|$ = √116 = 2√29. **1**

$a_2 - a_1$ = (4, 6, 8); $(a_2 - a_1) \cdot (b_1 \times b_2)$ = −16 − 36 − 64 = −116. **1**

SD = $\frac{|-116|}{2\sqrt{29}}=\frac{58}{\sqrt{29}}=2\sqrt{29}$ units. **1**

∴ **2√29 units**

OR

General points: (3λ − 1, 5λ − 3, 7λ − 5) and (μ + 2, 3μ + 4, 5μ + 6). **1**

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Equating x and y: $3\lambda - \mu = 3$ and $5\lambda - 3\mu = 7$ give $\lambda = 1/2$, $\mu = -3/2$; check z: $7(1/2) - 5 = -3/2$ and $5(-3/2) + 6 = -3/2$, so the lines intersect. **1**

Point of intersection: $(1/2, -1/2, -3/2)$. **1**

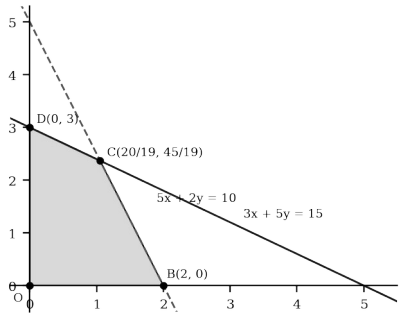
∴ **$(1/2, -1/2, -3/2)$**

31. Solution:

Draw $3x + 5y = 15$ and $5x + 2y = 10$; they meet where $x = 20/19$, $y = 45/19$. The feasible region is OBCD with O(0, 0), B(2, 0), C(20/19, 45/19), D(0, 3). **1**

Z at the corners: O: 0; B: 10; C: 100/19 + 135/19 = 235/19; D: 9. **1**

Maximum Z = 235/19 at $x = 20/19$, $y = 45/19$. **1**



∴ **Max Z = 235/19 at (20/19, 45/19)**

SECTION D

32. Solution:

Reflexive: $|a - a| = 0 = 4 \times 0$ for every $a \in A$, so $(a, a) \in R$. **1**

Symmetric: if $|a - b| = 4k$, then $|b - a| = |a - b| = 4k$, so $(b, a) \in R$. **1**

Transitive: if $a - b = \pm 4k$ and $b - c = \pm 4l$, then $a - c = (a - b) + (b - c)$ is a sum of multiples of 4, hence a multiple of 4; so $(a, c) \in R$. **1**

R is reflexive, symmetric and transitive, so it is an equivalence relation. **1**

Elements related to 1: those a with $|a - 1|$ a multiple of 4, i.e. {1, 5, 9}. **1**

∴ **Equivalence relation; {1, 5, 9}**

OR

For $x \geq 0$, $f(x) = x/(1 + x) \in [0, 1)$; for $x < 0$, $f(x) = x/(1 - x) \in (-1, 0)$. So f(x) has the same sign as x. **1**

One-one: if $f(x_1) = f(x_2)$, then x_1, x_2 have the same sign. For $x_1, x_2 \geq 0$: $x_1/(1 + x_1) = x_2/(1 + x_2)$ $\Rightarrow x_1 = x_2$; similarly for $x_1, x_2 < 0$. So f is one-one. **1**

Onto: take $y \in [0, 1)$; $x = y/(1 - y) \geq 0$ gives $f(x) = y$. **1**

For $y \in (-1, 0)$, $x = y/(1 + y) < 0$ gives $f(x) = y$. **1**

Every y in $(-1, 1)$ has a pre-image, so f is onto. Hence f is one-one and onto. **1**

∴ **One-one and onto**

33. Solution:

$x^3 - x = x(x - 1)(x + 1)$: it is ≥ 0 on $[-1, 0]$, ≤ 0 on $[0, 1]$ and ≥ 0 on $[1, 2]$. **1**

So the integral $= \int_{-1}^{0} (x^3 - x)\,dx + \int_{0}^{1} (x - x^3)\,dx + \int_{1}^{2} (x^3 - x)\,dx$. **1**

$\int_{-1}^{0}(x^3-x)\,dx = \left[\frac{x^4}{4}-\frac{x^2}{2}\right]$ from −1 to 0 = $0-\left(\frac{1}{4}-\frac{1}{2}\right)=\frac{1}{4}$. **1**

$\int_{0}^{1}(x-x^3)\,dx = \frac{1}{2}-\frac{1}{4}=\frac{1}{4}$; $\int_{1}^{2}(x^3-x)\,dx = (4-2)-\left(\frac{1}{4}-\frac{1}{2}\right)=\frac{9}{4}$. **1**

Total = $\frac{1}{4}+\frac{1}{4}+\frac{9}{4}=\frac{11}{4}$. **1**

∴ **11/4**

34. Solution:

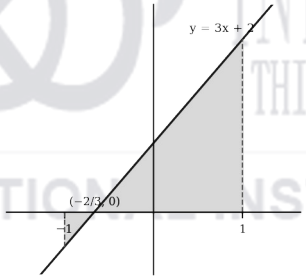
The line y = 3x + 2 meets the x-axis at x = −2/3. It lies below the x-axis on [−1, −2/3] and above it on [−2/3, 1] (see figure). **1**

Area = $\left|\int_{-1}^{-\frac{2}{3}}(3x+2)\,dx\right| + \int_{-\frac{2}{3}}^{1}(3x+2)\,dx$. **1**

$\left[\frac{3x^2}{2}+2x\right]$ from −1 to $-\frac{2}{3} = \left(\frac{2}{3}-\frac{4}{3}\right)-\left(\frac{3}{2}-2\right)=-\frac{1}{6}$, so the first part is $\frac{1}{6}$. **1**

$\left[\frac{3x^2}{2}+2x\right]$ from $-\frac{2}{3}$ to 1 = $\frac{7}{2}-\left(-\frac{2}{3}\right)=\frac{25}{6}$. **1**

Area = $\frac{1}{6}+\frac{25}{6}=\frac{26}{6}=\frac{13}{3}$ square units. **1**



∴ **13/3 sq. units**

OR

|x + 3| = −(x + 3) for x < −3 and x + 3 for x ≥ −3; the graph is V-shaped with vertex (−3, 0), above the x-axis (see figure). **1**

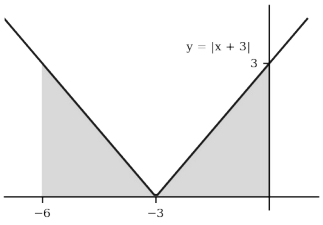
Area = $\int_{-6}^{-3}-(x+3)\,dx + \int_{-3}^{0}(x+3)\,dx$. **1**

$\int_{-6}^{-3}-(x+3)\,dx = -\left[\frac{x^2}{2}+3x\right]$ from −6 to $-3 = -\left(-\frac{9}{2}-0\right)=\frac{9}{2}$. **1**

$\int_{-3}^{0}(x+3)\,dx = \left[\frac{x^2}{2}+3x\right]$ from −3 to $0 = 0-\left(-\frac{9}{2}\right)=\frac{9}{2}$. **1**

Area = 9/2 + 9/2 = 9 square units. **1**

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∴ **9 sq. units**

35. Solution:

Let E_1: she gets 5 or 6, E_2: she gets 1, 2, 3 or 4, and A: exactly one head. $P(E_1) = 1/3$, $P(E_2) = 2/3$. **1**

$P(A|E_1)$ = P(exactly one head in 3 tosses) = 3/8; $P(A|E_2) = 1/2$. (a) $P(A) = (1/3)(3/8) + (2/3)(1/2) = 1/8 + 1/3 = 11/24$. **1**

(b) Bayes' theorem: $P(E_2|A) = \dfrac{\frac{2}{3}\times\frac{1}{2}}{\frac{11}{24}} = \dfrac{\frac{1}{3}}{\frac{11}{24}}$. **1**

$= \dfrac{8}{11}$. **1**

(c) $P(E_1|A) = 1 - \dfrac{8}{11} = \dfrac{3}{11}$. **1**

∴ **(a) 11/24 (b) 8/11 (c) 3/11**

SECTION E

36. Solution:

(i)

L_1: 2, 3, 4; L_2: 1, 2, 1. **1**

(ii)

$\dfrac{2}{1} \neq \dfrac{3}{2}$, so the direction ratios are not proportional: the beams are not parallel. **1**

(iii)(a)

General points: (1 + 2s, −1 + 3s, 1 + 4s) and (3 + t, k + 2t, t). Equating x and z: 1 + 2s = 3 + t and 1 + 4s = t give s = −3/2, t = −5. **1**

Equating y: −1 + 3(−3/2) = k + 2(−5) ⇒ −11/2 = k − 10 ⇒ k = 9/2. **1**

∴ **k = 9/2**

OR

(iii)(b)

$\cos\theta = \dfrac{|2(1)+3(2)+4(1)|}{\sqrt{29}\times\sqrt{6}} = \dfrac{12}{\sqrt{174}}$. **1**

$\theta = \cos^{-1}\left(\dfrac{12}{\sqrt{174}}\right)$. **1**

∴ **$\cos^{-1}(12/\sqrt{174})$**

37. Solution:

(i)

$xy = 24 \Rightarrow y = \frac{24}{x}$; the card measures (x + 3) by (y + 2), so $A = (x + 3)\left(\frac{24}{x} + 2\right) = 30 + 2x + \frac{72}{x}$. **1**

(ii)

$\frac{dA}{dx} = 2 - \frac{72}{x^2}$. **1**

(iii)(a)

$\frac{dA}{dx} = 0 \Rightarrow x^2 = 36 \Rightarrow x = 6, y = 4$; $\frac{d^2A}{dx^2} = \frac{144}{x^3} > 0$, so A is minimum. **1**

Card: (6 + 3) cm × (4 + 2) cm = 9 cm × 6 cm. **1**

∴ **9 cm × 6 cm**

OR

(iii)(b)

At x = 6, $\frac{d^2A}{dx^2} = \frac{144}{216} > 0$, so A is minimum there. **1**

Minimum area = 30 + 12 + 12 = 54 cm². **1**

∴ **54 cm²**

38. Solution:

(i)

$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 250 \\ 230 \\ 210 \end{bmatrix}$. **1**

$|A| = 3(4 - 3) - 2(2 - 6) + 1(1 - 4) = 3 + 8 - 3 = 8$. **1**

∴ **|A| = 8**

(ii)

$|A| \neq 0$, so $X = A^{-1}B = (1/|A|)(\text{adj } A)B = \frac{1}{8}\begin{bmatrix} 250 - 690 + 840 \\ 1000 + 920 - 1680 \\ -750 + 230 + 840 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 400 \\ 240 \\ 320 \end{bmatrix}$. **1**

x = 50, y = 30, z = 40: Magic ₹50, Puppet ₹30, Science ₹40. **1**

∴ **₹50, ₹30, ₹40**

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