MARKING SCHEME

MATHEMATICS (041)

Sample Paper 03 • Class XII • Session 2026–27

Q.P. Code : MC/SP/03 **Maximum Marks : 80**

Step-by-step solutions with the value points and marks for each step, as per the CBSE marking scheme. Marks for each step are shown in the right-hand column.

SECTION A

1. **Correct answer: (B) $\frac{7\pi}{18}$** **1**

$\cos\frac{\pi}{9} = \sin\left(\frac{\pi}{2} - \frac{\pi}{9}\right) = \sin\frac{7\pi}{18}$.

$\frac{7\pi}{18}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the principal value branch of $\sin^{-1}$, so the value is $\frac{7\pi}{18}$.

2. **Correct answer: (D) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$** **1**

cosec is one-one on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and takes every value in R − (−1, 1) there.

0 is excluded because cosec 0 is not defined.

3. **Correct answer: (A) $\frac{4}{3}$** **1**

Let $\sin^{-1}\frac{3}{5} = \theta$, so $\sin\theta = \frac{3}{5}$ with $\theta \in \left(0, \frac{\pi}{2}\right)$; then $\cos\theta = \frac{4}{5}$.

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{4}{3}$.

4. **Correct answer: (C) −6, −4, −9** **1**

$kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$; comparing the (2, 2) entries, $-4k = 24 \Rightarrow k = -6$.

Then $3a = 2k = -12 \Rightarrow a = -4$ and $2b = 3k = -18 \Rightarrow b = -9$.

5. **Correct answer: (B) 24** **1**

For a square matrix of order n, $|kA| = k^n|A|$.

$|2A| = 2^3 \times 3 = 24$.

6. **Correct answer: (D) skew-symmetric matrix** **1**

All diagonal entries are 0 and each $a_{ji} = -a_{ij}$ (e.g. $a_{12} = -5$, $a_{21} = 5$).

So $A' = -A$: the matrix is skew-symmetric.

7. **Correct answer: (C) continuous everywhere, but not differentiable at x = 0 and x = 1** **1**

The sum of two continuous functions is continuous, so f is continuous everywhere.

Each modulus term has a corner at its zero, so f is not differentiable at x = 0 and x = 1.

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8. **Correct answer: (A) x $\frac{dy}{dx}$** **1**

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$, i.e. $\sqrt{1-x^2}\,\frac{dy}{dx} = 1$.

Differentiating: $\sqrt{1-x^2}\,\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\,\frac{dy}{dx} = 0 \Rightarrow (1 - x^2)\,\frac{d^2y}{dx^2} = x\,\frac{dy}{dx}$.

9. **Correct answer: (A) 1** **1**

$V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$; $\frac{dV}{dS} = \frac{dV/dr}{dS/dr} = \frac{4\pi r^2}{8\pi r} = \frac{r}{2}$.

At r = 2 cm, dV/dS = 1.

10. **Correct answer: (C) increasing on R** **1**

$f'(x) = 3x^2 - 6x + 3 = 3(x - 1)^2 \geq 0$ for all x, with equality only at x = 1.

Hence f is increasing on R.

11. **Correct answer: (D) 2** **1**

Squaring to make it a polynomial in the derivatives: $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$.

The highest order derivative $\frac{d^2y}{dx^2}$ appears with power 2, so the degree is 2.

12. **Correct answer: (B) $\frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + C$** **1**

e^(x log a) = a^x and e^(a log x) = x^a.

$\int (a^x + x^a)\,dx = \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + C$.

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13. **Correct answer: (D) $\frac{7}{\sqrt{5}}(\hat{i} - 2\hat{j})$** **1**

Unit vector along $\vec{a}$: $\frac{1}{\sqrt{5}}(\hat{i} - 2\hat{j})$, since $|\vec{a}| = \sqrt{5}$.

Required vector = 7 × unit vector = $\frac{7}{\sqrt{5}}(\hat{i} - 2\hat{j})$.

14. **Correct answer: (A) $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$** **1**

For non-zero vectors, $\vec{a} \cdot \vec{b} = 0$ means they are perpendicular, and $\vec{a} \times \vec{b} = \vec{0}$ means they are parallel.

Two non-zero vectors cannot be both, so at least one of them is the zero vector.

15. **Correct answer: (B) $\frac{2\pi}{3}$** **1**

$\cos\theta = \frac{(\hat{i} - \hat{j}) \cdot (\hat{j} - \hat{k})}{\sqrt{2} \times \sqrt{2}} = \frac{-1}{2}$.

$\theta = \frac{2\pi}{3}$.

16. **Correct answer: (C) $\frac{5}{9}$** **1**

$P(A' \cap B) = P(B) - P(A \cap B) = \frac{9}{13} - \frac{4}{13} = \frac{5}{13}$.

$P(A'|B) = \frac{\frac{5}{13}}{\frac{9}{13}} = \frac{5}{9}$.

17. **Correct answer: (B) $\frac{5}{14}$** **1**

P(first red) = $\frac{5}{8}$; given that, P(second red) = $\frac{4}{7}$ (multiplication theorem).

P(both red) = $\frac{5}{8} \times \frac{4}{7} = \frac{5}{14}$.

18. **Correct answer: (D) $\frac{4}{3}$** **1**

$k + 2k + 3k = 1 \Rightarrow k = \frac{1}{6}$.

Mean = $0 \times \frac{1}{6} + 1 \times \frac{2}{6} + 2 \times \frac{3}{6} = \frac{8}{6} = \frac{4}{3}$.

19. **Correct answer: (A)** **1**

R is a standard result — R is true.

With $f(x) = \frac{1}{x}$, $f'(x) = -\frac{1}{x^2}$, so by R the integral is $\frac{e^x}{x}$ + C. A is true and R explains it.

20. **Correct answer: (A)** **1**

R is the standard fact about general solutions — R is true.

By R, an equation of order 3 has 3 arbitrary constants in its general solution. A is true and follows from R.

SECTION B

21. **Solution:**

$\tan^2 x = \sec^2 x - 1$, so the integral = [tan x − x] from 0 to $\frac{\pi}{4}$. **1**

$= \left(1 - \frac{\pi}{4}\right) - 0 = 1 - \frac{\pi}{4}$. **1**

∴ **$1 - \pi/4$**

OR

$x^2 - 6x + 13 = (x - 3)^2 + 2^2$ (completing the square). **1**

$\int \frac{dx}{(x-3)^2 + 2^2} = \frac{1}{2} \tan^{-1}\frac{x-3}{2} + C$. **1**

∴ **$(1/2) \tan^{-1}((x - 3)/2) + C$**

22. Solution:

$e^y = \frac{1}{x+1} \Rightarrow y = -\log(x+1)$, so $\frac{dy}{dx} = -\frac{1}{x+1}$. **1**

$\frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} = \left(-\frac{1}{x+1}\right)^2 = \left(\frac{dy}{dx}\right)^2$. Hence shown. **1**

∴ **Shown**

23. Solution:

(A) Three Dimensional Geometry

Standard form: $\frac{x-2}{2} = \frac{y-\frac{5}{2}}{-\frac{3}{2}} = \frac{z+1}{1}$; direction ratios 2, $-\frac{3}{2}$, 1, i.e. 4, −3, 2. **1**

$\sqrt{16+9+4} = \sqrt{29}$, so the direction cosines are $\frac{4}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{2}{\sqrt{29}}$. **1**

∴ **4/√29, −3/√29, 2/√29**

OR

(B) Vector Algebra

$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} = 4 + 9 - 8 = 5$. **1**

$|\vec{a} - \vec{b}| = \sqrt{5}$. **1**

∴ **√5**

24. Solution:

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = (1+4)\hat{i} - (3-4)\hat{j} + (-3-1)\hat{k} = 5\hat{i} + \hat{j} - 4\hat{k}$. **1**

Area $= |\vec{a} \times \vec{b}| = \sqrt{25+1+16} = \sqrt{42}$ square units. **1**

∴ **√42 sq. units**

25. Solution:

The feasible region is the rectangle with corner points (0, 0), (3, 0), (3, 2) and (0, 2). **1**

Z = 0, 33, 47, 14 at these points; the maximum value of Z is 47 at (3, 2). **1**

∴ **Max Z = 47 at (3, 2)**

SECTION C

26. Solution:

$A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$. Any square matrix A = P + Q, where P = ½(A + A′) is symmetric and Q = ½(A − A′) is skew-symmetric. **1**

$P = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix}$; P′ = P, so P is symmetric. **1**

$Q = \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$; $Q' = -Q$, so Q is skew-symmetric, and A = P + Q. **1**

∴ **A = P + Q as above**

27. Solution:

$f'(x) = 6x^2 - 30x + 36 = 6(x - 2)(x - 3) = 0 \Rightarrow x = 2, 3$ (both in [1, 5]). **1**

f(1) = 24, f(2) = 29, f(3) = 28, f(5) = 56. **1**

Absolute maximum = 56 at x = 5; absolute minimum = 24 at x = 1. **1**

∴ **Max 56 at x = 5; Min 24 at x = 1**

28. Solution:

For $x \neq 0$, $f(x) = \frac{2\sin^2(kx/2)}{x\sin x} = \frac{k^2}{2} \times \left(\frac{\sin(kx/2)}{kx/2}\right)^2 \times \frac{x}{\sin x}$. **1**

As $x \to 0$, $f(x) \to \frac{k^2}{2}$. **1**

Continuity at 0: $\frac{k^2}{2} = \frac{1}{2} \Rightarrow k = \pm 1$. **1**

∴ **k = ±1**

OR

$\frac{dx}{d\theta} = a \sec\theta \tan\theta$, $\frac{dy}{d\theta} = b \sec^2\theta$, so $\frac{dy}{dx} = \frac{b\sec\theta}{a\tan\theta} = \frac{b}{a}\operatorname{cosec}\theta$. **1**

$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{b}{a}\operatorname{cosec}\theta\right) \times \frac{d\theta}{dx} = \frac{-b\operatorname{cosec}\theta\cot\theta}{a \cdot a\sec\theta\tan\theta} = -\frac{b}{a^2}\cot^3\theta$. **1**

At $\theta = \frac{\pi}{6}$: $\cot\frac{\pi}{6} = \sqrt{3}$, so $\frac{d^2y}{dx^2} = -\frac{3\sqrt{3}b}{a^2}$. **1**

∴ **$-3\sqrt{3}\,b/a^2$**

29. Solution:

Linear form with P = −1, Q = cos x; IF = e^(−∫dx) = e^{-x}. **1**

$y\,e^{-x} = \int e^{-x}\cos x\,dx = \frac{e^{-x}}{2}(\sin x - \cos x) + C$ (integrating by parts twice). **1**

$y = \frac{1}{2}(\sin x - \cos x) + C\,e^x$. **1**

∴ **$y = \frac{1}{2}(\sin x - \cos x) + Ce^x$**

OR

Separating the variables: $\frac{dy}{1+y^2} = -\frac{e^x\,dx}{1+e^{2x}}$. **1**

Integrating (put $t = e^x$ on the right): $\tan^{-1}y = -\tan^{-1}(e^x) + C$. **1**

At x = 0, y = 1: $\frac{\pi}{4} = -\frac{\pi}{4} + C \Rightarrow C = \frac{\pi}{2}$; so $\tan^{-1}y + \tan^{-1}(e^x) = \frac{\pi}{2}$. **1**

∴ **$\tan^{-1}y + \tan^{-1}(e^x) = \pi/2$**

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30. Solution:

Any point of the line: Q(2t − 1, 3 − 2t, −t); PQ = (2t − 2, 1 − 2t, 3 − t). **1**

PQ ⊥ line: 2(2t − 2) − 2(1 − 2t) − 1(3 − t) = 9t − 9 = 0 ⇒ t = 1; foot of the perpendicular Q(1, 1, −1). **1**

Length PQ = $\sqrt{(0^2 + 1^2 + 2^2)} = \sqrt{5}$ units. **1**

∴ **Foot (1, 1, −1); length $\sqrt{5}$ units**

OR

The lines are parallel, since $4\hat{i} - 2\hat{j} + 2\hat{k} = 2(2\hat{i} - \hat{j} + \hat{k})$; take $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$, $|\vec{b}| = \sqrt{6}$. **1**

$a_2 - a_1 = \hat{i} - \hat{k}$; $\vec{b} \times (a_2 - a_1) = \hat{i} + 3\hat{j} + \hat{k}$, with magnitude $\sqrt{11}$. **1**

SD = $\frac{|\vec{b} \times (a_2 - a_1)|}{|\vec{b}|} = \frac{\sqrt{11}}{\sqrt{6}} = \frac{\sqrt{66}}{6}$ units. **1**

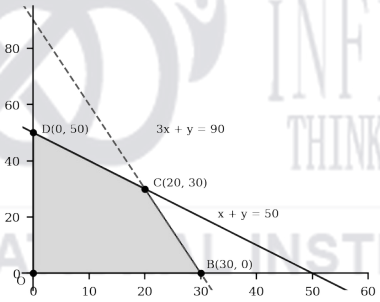
∴ **$\sqrt{66}/6$ units**

31. Solution:

Draw x + y = 50 and 3x + y = 90; they meet at (20, 30). The feasible region is OBCD with O(0, 0), B(30, 0), C(20, 30), D(0, 50). **1**

Z at the corners: O: 0; B: 120; C: 110; D: 50. **1**

Maximum Z = 120 at x = 30, y = 0. **1**



∴ **Max Z = 120 at (30, 0)**

SECTION D

32. Solution:

Reflexive: every triangle is similar to itself, so (T, T) ∈ R. **1**

Symmetric: if T_1 is similar to T_2, then T_2 is similar to T_1; so $(T_1, T_2) \in R \Rightarrow (T_2, T_1) \in R$. **1**

Transitive: if $T_1 \sim T_2$ and $T_2 \sim T_3$, then $T_1 \sim T_3$ (corresponding angles equal). So R is an equivalence relation. **1**

Similar triangles have proportional sides: 3/6 = 4/8 = 5/10 = 1/2, so $T_1 \sim T_3$. **1**

5/3, 12/4, 13/5 are not equal, so T_2 is similar to neither. Hence only T_1 and T_3 are related. **1**

∴ **Equivalence relation; T_1 and T_3 are related**

OR

If n is odd, f(n) = n + 1 is even; if n is even, f(n) = n − 1 is odd. **1**

One-one: if f(m) = f(n), then m and n are both odd or both even (an odd and an even number give images of different parity). **1**

Both odd: m + 1 = n + 1 ⇒ m = n; both even: m − 1 = n − 1 ⇒ m = n. So f is one-one. **1**

Onto: for an odd $k \in N$, $k + 1$ is even and $f(k + 1) = k$; for an even k, $k - 1$ is odd and $f(k - 1) = k$. **1**

Every $k \in N$ has a pre-image, so f is onto. Hence f is bijective. **1**

∴ **f is bijective**

33. Solution:

Let I be the integral. Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ (sin and cos interchange): $I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right)\sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$. **1**

Adding: $2I = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$. Divide numerator and denominator by $\cos^4 x$: the integrand becomes $\frac{\tan x \sec^2 x}{1 + \tan^4 x}$. **1**

Put $t = \tan^2 x$, $dt = 2 \tan x \sec^2 x\,dx$: $\int_0^{\frac{\pi}{2}} \frac{\tan x \sec^2 x}{1 + \tan^4 x}\,dx = \frac{1}{2}\int_0^{\infty} \frac{dt}{1 + t^2}$. **1**

$= \frac{1}{2}[\tan^{-1} t]$ from 0 to $\infty = \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$. **1**

$2I = \frac{\pi}{2} \times \frac{\pi}{4} \Rightarrow I = \frac{\pi^2}{16}$. **1**

∴ **$\pi^2/16$**

34. Solution:

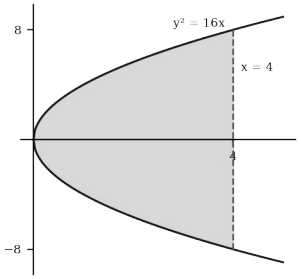
For $y^2 = 16x$, $4a = 16 \Rightarrow a = 4$, so the focus is (4, 0) and the latus rectum is the line $x = 4$. **1**

The region is symmetric about the x-axis; in the first quadrant the curve is $y = 4\sqrt{x}$ (see figure). **1**

Area $= 2\int_0^4 4\sqrt{x}\,dx$. **1**

$= 8 \times \frac{2}{3}$[x^(3/2)] from 0 to $4 = \frac{16}{3} \times 8$. **1**

$= \frac{128}{3}$ square units. **1**



∴ **128/3 sq. units**

OR

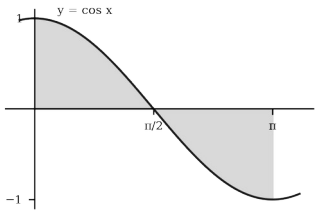
$\cos x \geq 0$ on $\left[0, \frac{\pi}{2}\right]$ and $\cos x \leq 0$ on $\left[\frac{\pi}{2}, \pi\right]$ (see figure). **1**

Area = $\int_0^{\frac{\pi}{2}} \cos x\, dx + \left|\int_{\frac{\pi}{2}}^{\pi} \cos x\, dx\right|$. **1**

$\int_0^{\frac{\pi}{2}} \cos x\, dx$ = [sin x] from 0 to $\frac{\pi}{2}$ = 1. **1**

$\int_{\frac{\pi}{2}}^{\pi} \cos x\, dx$ = [sin x] from $\frac{\pi}{2}$ to π = −1, so its absolute value is 1. **1**

Area = 1 + 1 = 2 square units. **1**



∴ **2 sq. units**

35. Solution:

Let E_1, E_2, E_3: loan at fixed, floating, variable rate; A: customer defaults. $P(E_1) = 0.1$, $P(E_2) = 0.2$, $P(E_3) = 0.7$; $P(A|E_1) = 0.05$, $P(A|E_2) = 0.03$, $P(A|E_3) = 0.01$. **1**

(a) By the theorem of total probability, $P(A) = 0.1 \times 0.05 + 0.2 \times 0.03 + 0.7 \times 0.01 = 0.005 + 0.006 + 0.007 = 0.018$. **1**

(b) By Bayes' theorem, $P(E_3|A) = \frac{0.7 \times 0.01}{0.018} = \frac{0.007}{0.018}$. **1**

$= \frac{7}{18}$. **1**

(c) $P(E_1|A) = \frac{0.005}{0.018} = \frac{5}{18}$. **1**

∴ **(a) 0.018 (b) 7/18 (c) 5/18**

SECTION E

36. Solution:

(i)

$\overrightarrow{AB} = (5 - 3)\hat{i} + (3 - 4)\hat{j} + (3 - 0)\hat{k}$; direction ratios 2, −1, 3. **1**

(ii)

$\overrightarrow{CD} = 7\hat{i} - \hat{j} - 5\hat{k}$, $|\overrightarrow{CD}| = \sqrt{75} = 5\sqrt{3}$; unit vector = $\frac{1}{5\sqrt{3}}(7\hat{i} - \hat{j} - 5\hat{k})$. **1**

(iii)(a)

$\overrightarrow{AB} \cdot \overrightarrow{CD} = 14 + 1 - 15 = 0$. **1**

So $\cos\theta = 0$ and $\theta = \frac{\pi}{2}$: the vectors are perpendicular. **1**

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∴ **90°**

OR

(iii)(b)

$\overrightarrow{AB} \times \overrightarrow{CD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 7 & -1 & -5 \end{vmatrix} = (5 + 3)\hat{i} - (-10 - 21)\hat{j} + (-2 + 7)\hat{k}.$ **1**

$= 8\hat{i} + 31\hat{j} + 5\hat{k}$, which is perpendicular to both. **1**

∴ $\mathbf{8\hat{i} + 31\hat{j} + 5\hat{k}}$

37. Solution:

(i)

Area of sheet: $\pi r^2 + 2\pi rh = 75\pi \Rightarrow h = \frac{75 - r^2}{2r}$; $V = \pi r^2 h = \frac{\pi}{2}(75r - r^3)$. **1**

(ii)

$\frac{dV}{dr} = \frac{\pi}{2}(75 - 3r^2)$. **1**

(iii)(a)

$\frac{dV}{dr} = 0 \Rightarrow r^2 = 25 \Rightarrow r = 5$ cm. **1**

$\frac{d^2V}{dr^2} = -3\pi r < 0$ at r = 5, so V is maximum when r = 5 cm. **1**

∴ **r = 5 cm**

OR

(iii)(b)

For maximum volume r = 5 cm (from dV/dr = 0 with $d^2V/dr^2 < 0$). **1**

Then $h = \frac{75 - 25}{10} = 5$ cm = r, so the statement h > r is false. **1**

∴ **False (h = r)**

38. Solution:

(i)

$x + y + z = 45$, $-x + z = 8$, $x - 2y + z = 0$; so $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$. **1**

$|A| = 1(0 + 2) - 1(-1 - 1) + 1(2 - 0) = 2 + 2 + 2 = 6$. **1**

∴ $\mathbf{|A| = 6}$

(ii)

$|A| \neq 0$, so $X = A^{-1}B = (1/|A|)(\text{adj } A)B = \frac{1}{6}\begin{bmatrix} 90 - 24 + 0 \\ 90 + 0 + 0 \\ 90 + 24 + 0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 66 \\ 90 \\ 114 \end{bmatrix}$. **1**

x = 11, y = 15, z = 19: the products are 11, 15 and 19 tonnes. **1**

∴ **11 t, 15 t, 19 t**

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