MARKING SCHEME

MATHEMATICS (041)

Sample Paper 04 • Class XII • Session 2026–27

Q.P. Code : MC/SP/04 **Maximum Marks : 80**

Step-by-step solutions with the value points and marks for each step, as per the CBSE marking scheme. Marks for each step are shown in the right-hand column.

SECTION A

1. **Correct answer: (D) $\frac{2\pi}{3}$** **1**

The principal value branch of $\cot^{-1}$ is $(0, \pi)$, and $\cot^{-1}(-x) = \pi - \cot^{-1}x$.

$\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

2. **Correct answer: (A) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$** **1**

$y = \sin^{-1}x$ takes values in the principal value branch of $\sin^{-1}x$.

This branch is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

3. **Correct answer: (B) $-\frac{\pi}{2}$** **1**

$\tan^{-1}\sqrt{3} = \frac{\pi}{3}$ and $\cot^{-1}(-\sqrt{3}) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

Value $= \frac{\pi}{3} - \frac{5\pi}{6} = -\frac{\pi}{2}$.

4. **Correct answer: (C) m × n** **1**

AB' is defined $\Rightarrow$ B' has n rows $\Rightarrow$ B has n columns; let B be p × n, so B' is n × p.

$B'A$ is defined $\Rightarrow$ number of columns of B' (= p) equals number of rows of A (= m). So B is m × n.

5. **Correct answer: (A) $\lambda \neq -\frac{8}{5}$** **1**

$|A| = 2(6 - 5) - \lambda(0 - 5) - 3(0 - 2) = 8 + 5\lambda$.

A^{-1} exists iff $|A| \neq 0$, i.e. $\lambda \neq -\frac{8}{5}$.

6. **Correct answer: (B) $1 - \alpha^2 - \beta\gamma = 0$** **1**

$$A^2 = \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix}.$$

$A^2 = I \Rightarrow \alpha^2 + \beta\gamma = 1$, i.e. $1 - \alpha^2 - \beta\gamma = 0$.

7. **Correct answer: (D) $\frac{5}{3}$** **1**

$\lim(x \to 0) \frac{\sin 5x}{3x} = \frac{5}{3} \times \lim(x \to 0) \frac{\sin 5x}{5x} = \frac{5}{3}$.

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Continuity at 0 requires $k = f(0) = \frac{5}{3}$.

8. **Correct answer: (C) $-e^x \tan e^x$** **1**

By the chain rule, $\frac{dy}{dx} = \frac{1}{\cos e^x} \times (-\sin e^x) \times e^x$.

$= -e^x \tan e^x$.

9. **Correct answer: (C) 2** **1**

$f'(x) = 1 - \frac{1}{x^2} = 0 \Rightarrow x = 1$ ($x > 0$); $f''(x) = \frac{2}{x^3} > 0$ at $x = 1$.

Local minimum value $= f(1) = 2$.

10. **Correct answer: (D) ₹66** **1**

Marginal revenue $= R'(x) = 6x + 36$.

At $x = 5$, $R'(5) = 30 + 36 = ₹66$.

11. **Correct answer: (A) $\frac{d^2y}{dx^2} - y = 0$** **1**

$\frac{dy}{dx} = C_1e^x - C_2e^{-x}$ and $\frac{d^2y}{dx^2} = C_1e^x + C_2e^{-x} = y$.

Eliminating the two constants: $\frac{d^2y}{dx^2} - y = 0$.

12. **Correct answer: (B) $\log(10^x + x^{10}) + C$** **1**

The numerator is the derivative of the denominator: $d/dx(x^{10} + 10^x) = 10x^9 + 10^x \log 10$.

So the integral $= \log(10^x + x^{10}) + C$.

13. **Correct answer: (A) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$** **1**

$\overrightarrow{PQ} = 3\hat{i} + 3\hat{j} + 3\hat{k}$, with $|\overrightarrow{PQ}| = 3\sqrt{3}$.

Unit vector $= \frac{3(\hat{i} + \hat{j} + \hat{k})}{3\sqrt{3}} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$.

14. **Correct answer: (D) $0 \le \theta \le \frac{\pi}{2}$** **1**

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, where the magnitudes are positive.

So $\vec{a} \cdot \vec{b} \ge 0 \Leftrightarrow \cos\theta \ge 0 \Leftrightarrow 0 \le \theta \le \frac{\pi}{2}$.

15. **Correct answer: (B) $\hat{i} - \hat{j} + 2\hat{k}$** **1**

$\overrightarrow{PQ} = (3 - 2)\hat{i} + (-4 + 3)\hat{j} + (7 - 5)\hat{k}$.

$= \hat{i} - \hat{j} + 2\hat{k}$.

16. **Correct answer: (C) not defined** **1**

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ is defined only when $P(B) \neq 0$.

Here $P(B) = 0$, so $P(A|B)$ is not defined.

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17. **Correct answer: (A) $\frac{1}{36}$** **1**

The only even prime number is 2, so the favourable outcome is (2, 2) only.

P = $\frac{1}{36}$.

18. **Correct answer: (B) 1** **1**

X = 0, 1, 2 with probabilities $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$.

Mean = $0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4} = 1$.

19. **Correct answer: (A)** **1**

R is a standard property of definite integrals — R is true.

Let I be the integral; by R (x → 10 − x), $I = \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}}\,dx$. Adding, $2I = \int_2^8 1\,dx = 6 \Rightarrow I =$ 3. A is true and R explains it.

20. **Correct answer: (A)** **1**

R is the standard formula for the integrating factor — R is true.

Here P = $\frac{1}{x}$, so IF = e^(∫dx/x) = e^(log x) = x. A is true and R explains it.

SECTION B

21. **Solution:**

$5 - 4x - x^2 = 9 - (x+2)^2 = 3^2 - (x+2)^2$. **1**

$\int \frac{dx}{\sqrt{3^2-(x+2)^2}} = \sin^{-1}\left(\frac{x+2}{3}\right) + C$. **1**

∴ $\sin^{-1}((x+2)/3) + C$

OR

Put $x^2 + 1 = t$, $2x\,dx = dt$; $x = 0 \Rightarrow t = 1$, $x = 1 \Rightarrow t = 2$. **1**

$\int_0^1 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_1^2 \frac{dt}{t} = \frac{1}{2}[\log t]$ from 1 to 2 $= \frac{1}{2}\log 2$. **1**

∴ (1/2) log 2

22. **Solution:**

Let $y = \sin^2 x$ and $z = e^{\cos x}$. Then $\frac{dy}{dx} = 2\sin x \cos x$ and $\frac{dz}{dx} = -\sin x\, e^{\cos x}$. **1**

$\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{2\sin x\cos x}{-\sin x\, e^{\cos x}} = -\frac{2\cos x}{e^{\cos x}}$. **1**

∴ −2 cos x / e^(cos x)

23. **Solution:**

(A) Three Dimensional Geometry

Direction ratios of the lines: 7, −5, 1 and 1, 2, 3. **1**

$a_1a_2 + b_1b_2 + c_1c_2 = 7 - 10 + 3 = 0$, so the lines are perpendicular. **1**

∴ Perpendicular

OR

(B) Vector Algebra

$\vec{a} + \vec{b} = 6\hat{i} + 2\hat{j} - 8\hat{k}$ and $\vec{a} - \vec{b} = 4\hat{i} - 4\hat{j} + 2\hat{k}$. **1**

$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 24 - 8 - 16 = 0$, so the vectors are perpendicular. **1**

∴ Shown

24. Solution:

$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} = 1 + 1 - 2\cos\theta = 2(1 - \cos\theta)$. **1**

$= 4\sin^2\frac{\theta}{2} \Rightarrow |\vec{a} - \vec{b}| = 2\sin\frac{\theta}{2}\left(as\ 0 \le \frac{\theta}{2} \le \frac{\pi}{2}\right)$; hence $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$. **1**

∴ Proved

25. Solution:

Z at the corner points: (0, 0): 0; (5, 0): 15; (6, 5): −2; (6, 8): −14; (4, 10): −28; (0, 8): −32. **1**

Maximum Z = 15 at (5, 0); minimum Z = −32 at (0, 8). **1**

∴ Max 15 at (5, 0); Min −32 at (0, 8)

SECTION C

26. Solution:

$AB = \begin{bmatrix} 2-3 & -4+9 \\ 1+4 & -2-12 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$; $|AB| = 14 - 25 = -11$, so $(AB)^{-1} = \frac{1}{-11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$. **1**

$|A| = -11$, $A^{-1} = \frac{1}{11}\begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}$; $|B| = 1$, $B^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$. **1**

$B^{-1}A^{-1} = \frac{1}{11}\begin{bmatrix} 12+2 & 9-4 \\ 4+1 & 3-2 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix} = (AB)^{-1}$. Hence verified. **1**

∴ Verified

27. Solution:

$f'(x) = 3(x+1)^2(x-3)^3 + 3(x+1)^3(x-3)^2 = 3(x+1)^2(x-3)^2(2x-2) = 6(x+1)^2(x-3)^2(x-1)$. **1**

$f'(x) = 0 \Rightarrow x = -1, 1, 3$. Since $(x+1)^2(x-3)^2 \ge 0$, the sign of $f'(x)$ is the sign of $(x-1)$. **1**

$f'(x) < 0$ for $x < 1$ $(x \ne -1)$ and $f'(x) > 0$ for $x > 1$ $(x \ne 3)$: f is strictly decreasing on $(-\infty, 1]$ and strictly increasing on $[1, \infty)$. **1**

∴ Decreasing on $(-\infty, 1]$; increasing on $[1, \infty)$

28. Solution:

Put $x - a = c\cos\theta$, $y - b = c\sin\theta$. Then $\frac{dx}{d\theta} = -c\sin\theta$, $\frac{dy}{d\theta} = c\cos\theta$, so $\frac{dy}{dx} = -\cot\theta$. **1**

$\frac{d^2y}{dx^2} = \text{cosec}^2\theta \times \frac{d\theta}{dx} = -\frac{cosec^3\theta}{c}$. **1**

$\frac{[1+\cot^2\theta]^{(3/2)}}{-cosec^3\theta/c} = \frac{cosec^3\theta}{-cosec^3\theta/c}$ = −c, a constant independent of a and b. **1**

∴ Value = −c

OR

$x\sqrt{1+y} = -y\sqrt{1+x}$; squaring: $x^2(1 + y) = y^2(1 + x) \Rightarrow x^2 - y^2 = xy(y - x)$. **1**

$(x - y)(x + y) = -xy(x - y)$; since $x \neq y$, $x + y = -xy \Rightarrow y = -\frac{x}{1+x}$. **1**

$\frac{dy}{dx} = -\frac{(1+x)-x}{(1+x)^2} = -\frac{1}{(1+x)^2}$. Hence proved. **1**

∴ Proved

29. Solution:

Linear differential equation with P = 3, Q = e^{-2x}; IF = e^(∫3 dx) = e^{3x}. **1**

$y\,e^{3x} = \int e^{-2x} \times e^{3x}\,dx = \int e^x\,dx = e^x + C$. **1**

$y = e^{-2x} + C\,e^{-3x}$ is the general solution. **1**

∴ $y = e^{-2x} + Ce^{-3x}$

OR

Homogeneous: put $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$; then $v + x\frac{dv}{dx} = v - \tan v \Rightarrow \cot v\,dv = -\frac{dx}{x}$. **1**

Integrating: $\log|\sin v| = -\log|x| + \log C \Rightarrow x \sin\left(\frac{y}{x}\right) = C$. **1**

At x = 1, $y = \frac{\pi}{4}$: $C = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$; the particular solution is $x \sin\left(\frac{y}{x}\right) = \frac{1}{\sqrt{2}}$. **1**

∴ x sin(y/x) = 1/√2

30. Solution:

Any point on the line is Q(3t − 2, 2t − 1, 2t + 3). **1**

$PQ^2 = (3t - 3)^2 + (2t - 4)^2 + (2t)^2 = 17t^2 - 34t + 25$; $PQ = 5 \Rightarrow 17t^2 - 34t = 0 \Rightarrow t = 0$ or $t = 2$. **1**

The required points are (−2, −1, 3) and (4, 3, 7). **1**

∴ (−2, −1, 3) and (4, 3, 7)

OR

Standard forms: $\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{2}$ and $\frac{x-1}{-3p/7} = \frac{y-5}{1} = \frac{z-6}{-5}$. **1**

Direction ratios: −3, 2p/7, 2 and −3p/7, 1, −5. **1**

Perpendicular: $\frac{9p}{7} + \frac{2p}{7} - 10 = 0 \Rightarrow \frac{11p}{7} = 10 \Rightarrow p = \frac{70}{11}$. **1**

∴ p = 70/11

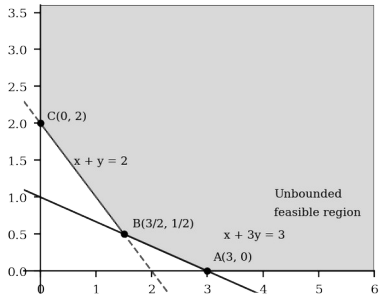
31. Solution:

Draw x + 3y = 3 and x + y = 2; they meet at B(3/2, 1/2). The feasible region is unbounded with corner points A(3, 0), B(3/2, 1/2) and C(0, 2). **1**

Z at the corners: A: 9; B: 7; C: 10. The smallest value is 7. **1**

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As the region is unbounded, draw $3x + 5y < 7$: it has no point in common with the feasible region. Hence minimum Z = 7 at x = 3/2, y = 1/2. **1**



∴ Min Z = 7 at (3/2, 1/2)

SECTION D

32. Solution:

Let O be the origin. Reflexive: OP = OP, so $(P, P) \in R$ for every point P. **1**

Symmetric: $(P, Q) \in R \Rightarrow OP = OQ \Rightarrow OQ = OP \Rightarrow (Q, P) \in R$. **1**

Transitive: $(P, Q), (Q, S) \in R \Rightarrow OP = OQ = OS \Rightarrow (P, S) \in R$. Hence R is an equivalence relation. **1**

Let $P \neq (0, 0)$ with $OP = r > 0$. A point Q is related to P iff $OQ = OP = r$. **1**

So the set of all points related to P is $\{Q : OQ = r\}$, the circle with centre O passing through P. **1**

∴ Equivalence relation; circle through P centred at O

OR

$f(x) = x^2$ for $x \geq 0$ and $f(x) = -x^2$ for $x < 0$; so f(x) has the same sign as x ($f(x) = 0$ only at $x = 0$). **1**

One-one: if $f(x_1) = f(x_2)$, then x_1 and x_2 have the same sign; for $x_1, x_2 \geq 0$, $x_1^2 = x_2^2 \Rightarrow x_1 = x_2$. **1**

For $x_1, x_2 < 0$, $-x_1^2 = -x_2^2 \Rightarrow x_1 = x_2$ (both negative). So f is one-one. **1**

Onto: for $y \geq 0$, take $x = \sqrt{y} \geq 0$: $f(x) = y$. For $y < 0$, take $x = -\sqrt{-y} < 0$: $f(x) = -(-y) = y$. **1**

Every real y has a pre-image, so f is onto. Hence f is bijective. **1**

∴ f is bijective

33. Solution:

Let $\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$, so $x^2 + x + 1 = A(x + 1)(x + 2) + B(x + 2) + C(x + 1)^2$. **1**

Put $x = -1$: $1 = B \Rightarrow B = 1$. Put $x = -2$: $3 = C \Rightarrow C = 3$. **1**

Comparing coefficients of x^2: $1 = A + C \Rightarrow A = -2$. **1**

$\int = -2\int\frac{dx}{x+1} + \int\frac{dx}{(x+1)^2} + 3\int\frac{dx}{x+2}$. **1**

$= -2\log|x + 1| - \frac{1}{x+1} + 3\log|x + 2| + C$. **1**

∴ $-2\log|x + 1| - 1/(x + 1) + 3\log|x + 2| + C$

34. Solution:

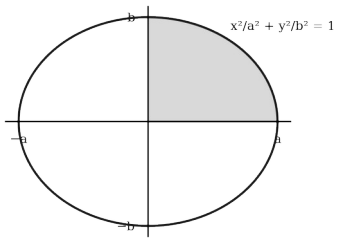
The ellipse is symmetric about both axes, so Area = 4 × (area in the first quadrant), where y = $\frac{b}{a}\sqrt{a^2 - x^2}$ (see figure). **1**

Area = $4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx$. **1**

= $\frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$ from 0 to a. **1**

= $\frac{4b}{a}\left[0 + \frac{a^2}{2} \times \frac{\pi}{2} - 0\right]$. **1**

= πab square units. **1**



∴ πab sq. units

OR

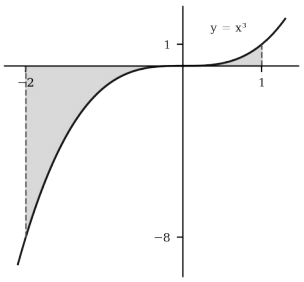
$y = x^3$ is negative for $x < 0$ and positive for $x > 0$, so the region has two parts (see figure). **1**

Area = $\left|\int_{-2}^{0} x^3\,dx\right| + \int_0^1 x^3\,dx$. **1**

$\int_{-2}^{0} x^3\,dx = \left[\frac{x^4}{4}\right]$ from −2 to 0 = −4, so the first part has area 4. **1**

$\int_0^1 x^3\,dx = \frac{1}{4}$. **1**

Area = $4 + \frac{1}{4} = \frac{17}{4}$ square units. **1**



∴ 17/4 sq. units

35. Solution:

Let E_1, E_2, E_3: smartphone made in A, B, C; D: it is defective. $P(E_1) = 0.5$, $P(E_2) = 0.3$, $P(E_3) = 0.2$; $P(D|E_1) = 0.02$, $P(D|E_2) = 0.03$, $P(D|E_3) = 0.04$. **1**

(a) By total probability, $P(D) = 0.5 \times 0.02 + 0.3 \times 0.03 + 0.2 \times 0.04 = 0.010 + 0.009 + 0.008 = 0.027$. **1**

(b) By Bayes' theorem, $P(E_1|D) = \frac{0.5 \times 0.02}{0.027} = \frac{0.010}{0.027}$. **1**

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$= \frac{10}{27}$. **1**

(c) $P(E_2|D) = \frac{0.009}{0.027} = \frac{1}{3}$. **1**

∴ (a) 0.027 (b) 10/27 (c) 1/3

SECTION E

36. Solution:

(i)

Direction vectors $\vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$; $2/3 \neq (-1)/(-5)$, so the tunnels are not parallel. **1**

∴ Not parallel

(ii)

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$. **1**

∴ $3\hat{i} - \hat{j} - 7\hat{k}$

(iii)(a)

$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$; $(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = 3 + 7 = 10$, and $|\vec{b}_1 \times \vec{b}_2| = \sqrt{59}$. **1**

SD $= \frac{10}{\sqrt{59}}$ units. **1**

∴ 10/√59 units

OR

(iii)(b)

$\cos\theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1||\vec{b}_2|} = \frac{6+5+2}{\sqrt{6}\sqrt{38}} = \frac{13}{\sqrt{228}}$. **1**

$\theta = \cos^{-1}\left(\frac{13}{2\sqrt{57}}\right)$. **1**

∴ $\cos^{-1}(13/(2\sqrt{57}))$

37. Solution:

(i)

$R(x) = px = 450x - \frac{x^2}{2}$. **1**

(ii)

$R'(x) = 450 - x$; at $x = 100$, marginal revenue = ₹350. **1**

∴ ₹350

(iii)(a)

Profit $P(x) = R(x) - C(x) = 400x - \frac{x^2}{2} - 10000$; $P'(x) = 400 - x = 0 \Rightarrow x = 400$, and $P''(x) = -1 < 0$. **1**

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Maximum profit = P(400) = 160000 − 80000 − 10000 = ₹70,000 when 400 calculators are sold. **1**

∴ 400 calculators; ₹70,000

OR

(iii)(b)

$R'(x) = 450 - x > 0 \Leftrightarrow x < 450$. **1**

R(x) is increasing on (0, 450) ($x \geq 0$ being the number of calculators). **1**

∴ (0, 450)

38. Solution:

(i)

$x + 2y + 3z = 320$, $2x + y + 2z = 250$, $3x + 2y + z = 280$; $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 320 \\ 250 \\ 280 \end{bmatrix}$. **1**

$|A| = 1(1 - 4) - 2(2 - 6) + 3(4 - 3) = -3 + 8 + 3 = 8$. **1**

∴ |A| = 8

(ii)

$X = A^{-1}B = (1/|A|)(\text{adj } A)B = \frac{1}{8}\begin{bmatrix} -960+1000+280 \\ 1280-2000+1120 \\ 320+1000-840 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 320 \\ 400 \\ 480 \end{bmatrix}$. **1**

$x = 40$, $y = 50$, $z = 60$: rice P costs ₹40, Q ₹50 and R ₹60 per kg. **1**

∴ ₹40, ₹50, ₹60 per kg

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