MARKING SCHEME

MATHEMATICS (041)

Sample Paper 05 • Class XII • Session 2026–27

Q.P. Code : MC/SP/05 **Maximum Marks : 80**

Step-by-step solutions with the value points and marks for each step, as per the CBSE marking scheme. Marks for each step are shown in the right-hand column.

SECTION A

1. **Correct answer: (A) $-\frac{\pi}{4}$** **1**

The principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

$\text{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}$ and $-\frac{\pi}{4}$ lies in this branch, so the value is $-\frac{\pi}{4}$.

2. **Correct answer: (B) $(-\infty, -1] \cup [0, \infty)$** **1**

$\sec^{-1}t$ is defined for $|t| \geq 1$, so $2x + 1 \leq -1$ or $2x + 1 \geq 1$.

$x \leq -1$ or $x \geq 0$: the domain is $(-\infty, -1] \cup [0, \infty)$.

3. **Correct answer: (C) $\frac{19}{8}$** **1**

Let $\cos^{-1}\frac{3}{5} = \theta$; then $\tan\theta = \frac{4}{3}$. Let $\tan^{-1}\frac{1}{4} = \varphi$, so $\tan\varphi = \frac{1}{4}$.

$$\tan(\theta + \varphi) = \frac{\frac{4}{3} + \frac{1}{4}}{1 - \frac{4}{3} \times \frac{1}{4}} = \frac{\frac{19}{12}}{\frac{2}{3}} = \frac{19}{8}.$$

4. **Correct answer: (D) I** **1**

$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$

So $A^{2026} = (A^2)^{1013} = I$.

5. **Correct answer: (C) 0** **1**

The first row is 6 times the third row: $(102, 18, 36) = 6(17, 3, 6)$.

Taking 6 common from R_1 makes $R_1 = R_3$, so the determinant is 0.

6. **Correct answer: (D) $\frac{\pi}{3}$** **1**

$A + A' = \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$

$2\cos\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}.$

7. **Correct answer: (A) 6** **1**

For $x \neq 3$, $f(x) = \dfrac{(x-3)(x+3)}{x-3} = x + 3$, so $\lim(x \to 3) f(x) = 6$.

Continuity at 3 requires $k = 6$.

8. **Correct answer: (B) $-\tan\theta$** **1**

$\dfrac{dx}{d\theta} = -3a\cos^2\theta \sin\theta$ and $\dfrac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$.

$\dfrac{dy}{dx} = \dfrac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$.

9. **Correct answer: (C) $\sqrt{2}$** **1**

$\sin x + \cos x = \sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right)$.

Since the maximum value of sin is 1, the maximum value is $\sqrt{2}\left(at\ x = \dfrac{\pi}{4}\right)$.

10. **Correct answer: (D) −2** **1**

$f'(x) = 2x + a$ must be ≥ 0 for all x in [1, 2]; its least value there is $2 + a$ (at $x = 1$).

$2 + a \geq 0 \Rightarrow a \geq -2$; the least value of a is −2.

11. **Correct answer: (A) 0** **1**

A particular solution is obtained from the general solution by giving particular values to the arbitrary constants.

So a particular solution has no arbitrary constants: the answer is 0.

12. **Correct answer: (B) e − 1** **1**

Put $\sin x = t$, $\cos x\, dx = dt$; $x = 0 \Rightarrow t = 0$ and $x = \pi/2 \Rightarrow t = 1$.

$\int_0^1 e^t\, dt = e - 1$.

13. **Correct answer: (A) $12\sqrt{3}$** **1**

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta \Rightarrow 12 = 24\sin\theta \Rightarrow \sin\theta = \dfrac{1}{2}$, so $\theta = \dfrac{\pi}{6}$ (acute).

$\vec{a} \cdot \vec{b} = 24\cos\dfrac{\pi}{6} = 12\sqrt{3}$.

14. **Correct answer: (C) 1, 2, 3** **1**

Dividing each part by 6: $\dfrac{x - \frac{1}{3}}{1} = \dfrac{y + \frac{1}{3}}{2} = \dfrac{z-1}{3}$.

So the direction ratios are 1, 2, 3.

15. **Correct answer: (B) $|\vec{a}| = |\vec{b}|$** **1**

$\vec{a} + \vec{b}$ is a diagonal of the parallelogram with sides $\vec{a}$ and $\vec{b}$.

A diagonal bisects the angle only when the parallelogram is a rhombus, i.e. $|a| = |b|$.

16. **Correct answer: (D) $\dfrac{5}{9}$** **1**

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$P(A|B) = \dfrac{\frac{1}{5}}{\frac{3}{10}} = \dfrac{2}{3}$; $P(A \cup B) = \dfrac{2}{5} + \dfrac{3}{10} - \dfrac{1}{5} = \dfrac{1}{2}$, so $P(A' \cap B') = \dfrac{1}{2}$.

$P(B'|A') = \dfrac{\frac{1}{2}}{\frac{3}{5}} = \dfrac{5}{6}$; product $= \dfrac{2}{3} \times \dfrac{5}{6} = \dfrac{5}{9}$.

17. **Correct answer: (A) $\dfrac{1}{6}$** **1**

First throw even: 18 outcomes; among them sum 8: (2, 6), (4, 4), (6, 2).

Required probability $= \dfrac{3}{18} = \dfrac{1}{6}$.

18. **Correct answer: (C) $\dfrac{7}{10}$** **1**

$k(1 + 2 + 3 + 4) = 1 \Rightarrow k = \dfrac{1}{10}$.

$P(X \geq 3) = 3k + 4k = \dfrac{7}{10}$.

19. **Correct answer: (A)** **1**

R is the standard result obtained by putting $f(x) = t$ — R is true.

With $f(x) = \log x$, $f'(x) = \dfrac{1}{x}$ and $n = 1$, R gives $\dfrac{(\log x)^2}{2} + C$. A is true and R explains it.

20. **Correct answer: (C)** **1**

For $y = e^{-3x}$: $9e^{-3x} - 3e^{-3x} - 6e^{-3x} = 0$, so A is true.

For $y = e^{3x}$: $9e^{3x} + 3e^{3x} - 6e^{3x} = 6e^{3x} \neq 0$, so R is false.

SECTION B

21. **Solution:**

Put $x^2 = t$, $2x\,dx = dt$; $x = 0 \Rightarrow t = 0$, $x = 1 \Rightarrow t = 1$. **1**

$\dfrac{1}{2}\int_0^1 e^t\,dt = \dfrac{1}{2}[e^t]$ from 0 to 1 $= \dfrac{e-1}{2}$. **1**

$\therefore$ **(e − 1)/2**

OR

$\sin 2x \cos 3x = \dfrac{1}{2}[\sin 5x + \sin(-x)] = \dfrac{1}{2}[\sin 5x - \sin x]$. **1**

$\int = \dfrac{1}{2}\left[-\dfrac{\cos 5x}{5} + \cos x\right] + C = -\dfrac{\cos 5x}{10} + \dfrac{\cos x}{2} + C$. **1**

$\therefore$ **−(cos 5x)/10 + (cos x)/2 + C**

22. **Solution:**

Taking logarithms: x log y = y log x. Differentiating: $\log y + \frac{x}{y}\frac{dy}{dx} = \frac{dy}{dx}\log x + \frac{y}{x}$. **1**

$\frac{dy}{dx}\left(\frac{x}{y} - \log x\right) = \frac{y}{x} - \log y \Rightarrow \frac{dy}{dx} = \frac{y(y - x\log y)}{x(x - y\log x)}$. **1**

∴ **y(y − x log y) / [x(x − y log x)]**

23. Solution:

(A) Three Dimensional Geometry

Direction ratios of AB: 1 − 2, −2 − 3, 3 + 4 = −1, −5, 7; of BC: 3 − 1, 8 + 2, −11 − 3 = 2, 10, −14. **1**

The direction ratios are proportional (BC = −2 × AB) and B is common, so A, B and C are collinear. **1**

∴ **Collinear**

OR

(B) Vector Algebra

$\vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$; $\vec{a} \cdot (\vec{b} + \vec{c}) = 6 - 2 + 2 = 6$. **1**

$|\vec{a}| = 3$; projection $= \frac{6}{3} = 2$. **1**

∴ **2**

24. Solution:

Let $|\vec{a}| = |\vec{b}| = |\vec{c}| = p$; then $|\vec{a} + \vec{b} + \vec{c}|^2 = 3p^2$ (all dot products between different vectors are 0), so the magnitude is $\sqrt{3}p$. **1**

If $\vec{a} + \vec{b} + \vec{c}$ makes angle α with $\vec{a}$: $\cos\alpha = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{\sqrt{3}\,p\cdot p} = \frac{p^2}{\sqrt{3}\,p^2} = \frac{1}{\sqrt{3}}$; similarly for $\vec{b}$ and $\vec{c}$. So it is equally inclined to all three. **1**

∴ **Shown**

25. Solution:

x + 2y = 8 and 3x + 2y = 12 meet at (2, 3); corner points: (0, 0), (4, 0), (2, 3), (0, 4). **1**

Z = 0, −12, 6, 16 respectively; minimum Z = −12 at (4, 0). **1**

∴ **Min Z = −12 at (4, 0)**

SECTION C

26. Solution:

(B′AB)′ = B′A′(B′)′ = B′A′B, using the reversal law (PQR)′ = R′Q′P′ and (B′)′ = B. **1**

If A is symmetric, A′ = A ⇒ (B′AB)′ = B′AB, so B′AB is symmetric. **1**

If A is skew-symmetric, A′ = −A ⇒ (B′AB)′ = −B′AB, so B′AB is skew-symmetric. **1**

∴ **Proved**

27. Solution:

f′(x) = 3x² − 12x + 9 = 3(x − 1)(x − 3) = 0 ⇒ x = 1, 3. **1**

f″(x) = 6x − 12: f″(1) = −6 < 0, so x = 1 is a point of local maximum; f″(3) = 6 > 0, so x = 3 is a point of local minimum. **1**

Local maximum value f(1) = 1 − 6 + 9 + 15 = 19; local minimum value f(3) = 27 − 54 + 27 + 15 = 15. **1**

∴ **Local max 19 at x = 1; local min 15 at x = 3**

28. Solution:

Let u = $(\cos x)^x$: log u = x log cos x ⇒ $\frac{1}{u}\frac{du}{dx}$ = log cos x − x tan x ⇒ $\frac{du}{dx}$ = $(\cos x)^x$(log cos x − x tan x). **1**

Let v = (sin x)^(1/x): log v = $\frac{\log \sin x}{x}$ ⇒ $\frac{1}{v}\frac{dv}{dx} = \frac{x\cot x - \log \sin x}{x^2}$. **1**

$\frac{dy}{dx}$ = $(\cos x)^x$(log cos x − x tan x) + (sin x)^(1/x) $\frac{x\cot x - \log \sin x}{x^2}$. **1**

∴ **As above**

OR

lim(x → 3⁻) |x − 3| = lim(x → 3⁺) |x − 3| = 0 = f(3), so f is continuous at x = 3. **1**

LHD = lim(h → 0) $\frac{|-h|-0}{-h}$ = −1 and RHD = lim(h → 0) $\frac{|h|-0}{h}$ = 1. **1**

LHD ≠ RHD, so f is not differentiable at x = 3. **1**

∴ **Continuous, not differentiable at x = 3**

29. Solution:

$\frac{dy}{dx} = 1 - 2\left(\frac{y}{x}\right)^2 + \frac{y}{x}$ is homogeneous; put y = vx: $v + x\frac{dv}{dx} = 1 - 2v^2 + v \Rightarrow \frac{dv}{1-2v^2} = \frac{dx}{x}$. **1**

$\frac{1}{2}\int\frac{dv}{\left(\frac{1}{\sqrt{2}}\right)^2 - v^2} = \log|x| + C \Rightarrow \frac{1}{2\sqrt{2}}\log\left|\frac{1+\sqrt{2}v}{1-\sqrt{2}v}\right| = \log|x| + C.$ **1**

Putting v = $\frac{y}{x}$: $\frac{1}{2\sqrt{2}}\log\left|\frac{x+\sqrt{2}y}{x-\sqrt{2}y}\right| = \log|x| + C.$ **1**

∴ **(1/(2√2)) log|(x + √2y)/(x − √2y)| = log|x| + C**

OR

Linear with P = 2, Q = sin x; IF = e^{2x}. **1**

y e^{2x} = ∫ e^{2x} sin x dx = $\frac{e^{2x}}{5}$(2 sin x − cos x) + C (integrating by parts twice). **1**

y = $\frac{1}{5}$(2 sin x − cos x) + C e^{-2x}. **1**

∴ **y = (2 sin x − cos x)/5 + Ce^{-2x}**

30. Solution:

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$ = 8î + 8ĵ + 4k̂, with magnitude 12. **1**

$\vec{a}_2 - \vec{a}_1$ = −10î − 2ĵ − 3k̂; $(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)$ = −80 − 16 − 12 = −108. **1**

SD = $\frac{|-108|}{12}$ = 9 units. **1**

∴ **9 units**

OR

Direction ratios: 2, 2, 1 and 4, 1, 8, with magnitudes 3 and 9. **1**

$\cos\theta = \frac{2\times4+2\times1+1\times8}{3\times9} = \frac{18}{27} = \frac{2}{3}$. **1**

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$. **1**

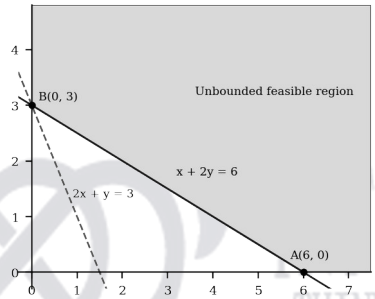
∴ **$\cos^{-1}(2/3)$**

31. Solution:

Draw 2x + y = 3 and x + 2y = 6. The feasible region is unbounded with corner points A(6, 0) and B(0, 3) (the lines meet on the y-axis at B). **1**

Z at A = 6 and Z at B = 6. **1**

The half-plane x + 2y < 6 has no point in common with the feasible region. So minimum Z = 6 at every point of the segment AB joining (6, 0) and (0, 3). **1**



∴ **Min Z = 6 at all points on the segment joining (6, 0) and (0, 3)**

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SECTION D

32. Solution:

Reflexive: for every real a, $1 + a\cdot a = 1 + a^2 > 0$, so $(a, a) \in R$. **1**

Symmetric: $(a, b) \in R \Rightarrow 1 + ab > 0 \Rightarrow 1 + ba > 0 \Rightarrow (b, a) \in R$. **1**

Not transitive: take $a = -1$, $b = \frac{1}{2}$, $c = 4$. Then $1 + ab = \frac{1}{2} > 0$, so $(a, b) \in R$. **1**

$1 + bc = 1 + 2 = 3 > 0$, so $(b, c) \in R$; but $1 + ac = 1 - 4 = -3 < 0$, so $(a, c) \notin R$. **1**

Hence R is reflexive and symmetric but not transitive. **1**

∴ **Shown**

OR

One-one: $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$. **1**

$-3x_1 - 2x_2 = -3x_2 - 2x_1 \Rightarrow x_1 = x_2$. So f is one-one. **1**

Onto: let $y \in B$ ($y \neq 1$). Solve $y = \frac{x-2}{x-3}$: $xy - 3y = x - 2 \Rightarrow x = \frac{3y-2}{y-1}$. **1**

x is real since $y \neq 1$, and $x \neq 3$ ($x = 3$ would give $3y - 3 = 3y - 2$, impossible); so $x \in A$. **1**

$f\left(\frac{3y-2}{y-1}\right) = y$, so every $y \in B$ has a pre-image: f is onto. **1**

∴ **f is one-one and onto**

33. Solution:

sin 2x = 2 sin x cos x; put sin x = t, cos x dx = dt; x = 0 ⇒ t = 0, x = $\frac{\pi}{2}$ ⇒ t = 1. **1**

I = $\int_0^1 2t \tan^{-1} t \, dt$. Integrating by parts with $\tan^{-1}$t as the first function: **1**

I = [$t^2 \tan^{-1}$t] from 0 to 1 − $\int_0^1 \frac{t^2}{1+t^2} dt = \frac{\pi}{4} - \int_0^1 \left(1 - \frac{1}{1+t^2}\right) dt$. **1**

= $\frac{\pi}{4}$ − [t − $\tan^{-1}$t] from 0 to 1 = $\frac{\pi}{4} - \left(1 - \frac{\pi}{4}\right)$. **1**

= $\frac{\pi}{2}$ − 1. **1**

∴ **π/2 − 1**

34. Solution:

$y = x^2 \geq 0$ for all x, so the region lies above the x-axis (see figure). **1**

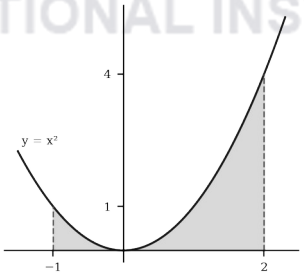
Area = $\int_{-1}^{2} x^2 dx$. **1**

= $\left[\frac{x^3}{3}\right]$ from −1 to 2. **1**

= $\frac{8}{3} - \left(-\frac{1}{3}\right) = \frac{9}{3}$. **1**

= 3 square units. **1**

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∴ **3 sq. units**

OR

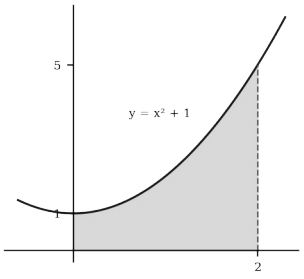
The region lies under the parabola $y = x^2 + 1$ (vertex (0, 1)), above the x-axis, between x = 0 and x = 2 (see figure). **1**

Area = $\int_0^2 (x^2+1) dx$. **1**

= $\left[\frac{x^3}{3} + x\right]$ from 0 to 2. **1**

= $\frac{8}{3}$ + 2. **1**

= $\frac{14}{3}$ square units. **1**



∴ **14/3 sq. units**

35. Solution:

(a) Sum of probabilities = 1: 0.1 + k + 2k + 2k + k = 1 ⇒ 6k = 0.9 ⇒ k = 0.15. **1**

(b) P(X ≥ 2) = 2k + 2k + k = 5k = 0.75. **1**

(c) Mean = ΣxP(x) = 0 × 0.1 + 1 × 0.15 + 2 × 0.30 + 3 × 0.30 + 4 × 0.15. **1**

= 0 + 0.15 + 0.60 + 0.90 + 0.60 = 2.25 cars. **1**

(d) $P(X = 4 \mid X \geq 2) = \frac{P(X=4)}{P(X \geq 2)} = \frac{0.15}{0.75} = \frac{1}{5}$. **1**

∴ **(a) 0.15 (b) 0.75 (c) 2.25 (d) 1/5**

SECTION E

36. Solution:

(i)

Direction ratios 1, −2, 1 with $\sqrt{1+4+1} = \sqrt{6}$; direction cosines $\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}$. **1**

(ii)

(5 − 3)/1 = 2, (−7 + 3)/(−2) = 2, (5 − 3)/1 = 2: all ratios are equal, so the point lies on L_1. **1**

∴ **Yes**

(iii)(a)

Any point of L_1: P(t + 3, −2t − 3, t + 3). OP ⊥ L_1: (t + 3) − 2(−2t − 3) + (t + 3) = 6t + 12 = 0 ⇒ t = −2. **1**

Foot of the perpendicular (1, 1, 1); distance = $\sqrt{1+1+1} = \sqrt{3}$ units. **1**

∴ **(1, 1, 1); √3 units**

OR

(iii)(b)

$\cos\theta = \frac{|1 \times 2 + (-2) \times 1 + 1 \times (-2)|}{\sqrt{6} \times 3} = \frac{2}{3\sqrt{6}}$. **1**

$\theta = \cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)$. **1**

∴ **$\cos^{-1}(2/(3\sqrt{6}))$**

37. Solution:

(i)

Time taken = $\frac{300}{v}$ h; $T = \frac{300}{v}\left(\frac{v^2}{25} + 144\right) = 12v + \frac{43200}{v}$. **1**

(ii)

$\frac{dT}{dv} = 12 - \frac{43200}{v^2}$. **1**

(iii)(a)

$\frac{dT}{dv} = 0 \Rightarrow v^2 = 3600 \Rightarrow v = 60$ km/h; $\frac{d^2T}{dv^2} = \frac{86400}{v^3} > 0$, so T is minimum. **1**

Minimum cost T(60) = 720 + 720 = ₹1440. **1**

∴ **60 km/h; ₹1440**

OR

(iii)(b)

At v = 60 km/h (from dT/dv = 0), time = 5 h; fuel cost = $5 \times \frac{3600}{25}$ = ₹720. **1**

Wages and other charges = 5 × 144 = ₹720. The two are equal. **1**

∴ **Both ₹720**

38. Solution:

(i)

$x + y + z = 50$, $6x + 8y + 9z = 405$, $x + y - z = 0$; $A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 8 & 9 \\ 1 & 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 50 \\ 405 \\ 0 \end{bmatrix}$. **1**

$|A| = 1(-8 - 9) - 1(-6 - 9) + 1(6 - 8) = -17 + 15 - 2 = -4$. **1**

∴ **$|A| = -4$**

(ii)

$X = A^{-1}B = (1/|A|)(\text{adj } A)B = \frac{1}{-4}\begin{bmatrix} -850+810+0 \\ 750-810+0 \\ -100+0+0 \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -40 \\ -60 \\ -100 \end{bmatrix}$. **1**

x = 10, y = 15, z = 25: ₹10,000 in A, ₹15,000 in B and ₹25,000 in C. **1**

∴ **₹10,000, ₹15,000, ₹25,000**

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