MARKING SCHEME

MATHEMATICS (041)

Sample Paper 06 • Class XII • Session 2026–27

Q.P. Code : MC/SP/06 **Maximum Marks : 80**

Step-by-step solutions with the value points and marks for each step, as per the CBSE marking scheme. Marks for each step are shown in the right-hand column.

SECTION A

1. **Correct answer: (C) $\frac{3\pi}{4}$** **1**

$\sec\frac{5\pi}{4} = -\sqrt{2}$; the principal value branch of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$.

$\sec\frac{3\pi}{4} = -\sqrt{2}$ and $\frac{3\pi}{4}$ lies in this branch, so the value is $\frac{3\pi}{4}$.

2. **Correct answer: (B) $\frac{5\pi}{6}$** **1**

$\sec^{-1}2 = \frac{\pi}{3}$ and $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

Value $= \frac{2\pi}{3} + \frac{\pi}{6} = \frac{5\pi}{6}$.

3. **Correct answer: (D) $\left[\frac{\pi}{2}, \pi\right]$** **1**

$\cos^{-1}x$ is decreasing on $[-1, 1]$ with $\cos^{-1}(-1) = \pi$ and $\cos^{-1}0 = \frac{\pi}{2}$.

So for $-1 \le x \le 0$, $\frac{\pi}{2} \le y \le \pi$.

4. **Correct answer: (A) 8** **1**

An m × n matrix has mn elements, so we need ordered pairs (m, n) of natural numbers with mn = 24.

24 has 8 divisors (1, 2, 3, 4, 6, 8, 12, 24), giving 8 possible orders.

5. **Correct answer: (D) 8** **1**

A is non-singular, so pre-multiplying $A^2 = 2A$ by A^{-1} gives $A = 2I$.

$|A| = |2I| = 2^3 \times 1 = 8$.

6. **Correct answer: (C) AB = BA** **1**

$(A + B)(A - B) = A^2 - AB + BA - B^2$.

This equals $A^2 - B^2$ iff $-AB + BA = O$, i.e. $AB = BA$.

7. **Correct answer: (B) continuous** **1**

$|x \sin\frac{1}{x}| \le |x| \to 0$ as $x \to 0$, so $\lim(x \to 0)\ f(x) = 0$.

This equals $f(0) = 0$, so f is continuous at x = 0.

8. **Correct answer: (A)** $\frac{\cos x}{2y-1}$ **1**

Squaring: $y^2 = \sin x + y$. Differentiating: $2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$.

$\frac{dy}{dx}(2y - 1) = \cos x \Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1}$.

9. **Correct answer: (A) 0.8 cm/s** **1**

Perimeter P = 4x, so $\frac{dP}{dt} = 4\frac{dx}{dt}$.

$= 4 \times 0.2 = 0.8$ cm/s.

10. **Correct answer: (B)** $\mathbf{x = \frac{1}{e}}$ **1**

$f'(x) = x^x(1 + \log x)$ (by logarithmic differentiation).

$f'(x) = 0 \Rightarrow \log x = -1 \Rightarrow x = \frac{1}{e}$.

11. **Correct answer: (D) 1** **1**

The family is $x^2 + y^2 = r^2$, with one arbitrary constant r.

Eliminating one constant gives a first order equation ($x + y\,dy/dx = 0$). Order = 1.

12. **Correct answer: (C)** $\mathbf{\frac{x^3}{3} + x + C}$ **1**

$x^3 - x^2 + x - 1 = x^2(x - 1) + (x - 1) = (x - 1)(x^2 + 1)$, so the integrand is $x^2 + 1$.

$\int(x^2 + 1)\,dx = \frac{x^3}{3} + x + C$.

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13. **Correct answer: (D)** $\sqrt{3}$ **1**

$\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$ and $\hat{k} \times \hat{i} = \hat{j}$.

$|\hat{i} + \hat{j} + \hat{k}| = \sqrt{3}$.

14. **Correct answer: (A)** $\vec{a} \perp \vec{b}$ **1**

Squaring: $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$.

$4\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}$.

15. **Correct answer: (C) coincident** **1**

Direction ratios 1, 2, 3 and −2, −4, −6 are proportional, so the lines are parallel.

The point (1, 2, 3) of the second line lies on the first (1/1 = 2/2 = 3/3), so the lines are coincident.

16. **Correct answer: (B)** $\frac{P(A)}{P(B)}$ **1**

$A \subset B \Rightarrow A \cap B = A$.

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$.

17. **Correct answer: (B)** $\frac{3}{7}$ **1**

P(at least one head) = $1 - \frac{1}{8} = \frac{7}{8}$; P(exactly two heads) = $\frac{3}{8}$ (HHT, HTH, THH).

Required probability = $\frac{\frac{3}{8}}{\frac{7}{8}} = \frac{3}{7}$.

18. **Correct answer: (D) $\frac{5}{18}$** **1**

Exactly one six: six on the first die only or on the second only: $2 \times \frac{1}{6} \times \frac{5}{6}$.

$= \frac{10}{36} = \frac{5}{18}$.

19. **Correct answer: (A)** **1**

R is the rule of integration by parts — R is true.

With u = x, v = sin x: ∫ x sin x dx = x(−cos x) − ∫(−cos x) dx = −x cos x + sin x + C. A is true and R explains it.

20. **Correct answer: (A)** **1**

R describes exactly the variables separable form — R is true.

$\frac{dy}{dx} = y(x^2 + 1) = g(x)h(y)$ with $g(x) = x^2 + 1$, $h(y) = y$; so $\frac{dy}{y} = (x^2 + 1)dx$. A is true and R explains it.

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SECTION B

21. **Solution:**

Multiply numerator and denominator by x^3 and put $x^4 = t$, $4x^3\,dx = dt$: $\int \frac{x^3\,dx}{x^4(x^4+1)} = \frac{1}{4}\int$ **1**

$\frac{dt}{t(t+1)} = \frac{1}{4}\int\left(\frac{1}{t} - \frac{1}{t+1}\right) dt.$

$= \frac{1}{4}\log\left|\frac{t}{t+1}\right| + C = \frac{1}{4}\log\left(\frac{x^4}{x^4+1}\right) + C.$ **1**

∴ **(1/4) log(x⁴/(x⁴ + 1)) + C**

OR

d/dx(x + cos x) = 1 − sin x, which is the numerator. **1**

∫ = log|x + cos x| + C. **1**

∴ **log|x + cos x| + C**

22. **Solution:**

$\log y = \sin x \log x \Rightarrow \frac{1}{y}\frac{dy}{dx} = \cos x \log x + \frac{\sin x}{x}$. **1**

$\frac{dy}{dx} = x^{\wedge}(\sin x)\left(\cos x \log x + \frac{\sin x}{x}\right)$. **1**

∴ **x^(sin x)(cos x log x + (sin x)/x)**

23. Solution:

(A) Three Dimensional Geometry

Direction ratios of AB: 3 − 1, 1 + 2, −3 − 3 = 2, 3, −6; √(4 + 9 + 36) = 7. **1**

Direction cosines: $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$. **1**

∴ **2/7, 3/7, −6/7**

OR

(B) Vector Algebra

$|\vec{a}|^2 - |\vec{b}|^2 = 8 \Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8 \Rightarrow |\vec{b}|^2 = \frac{8}{63}$. **1**

$|\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$ and $|\vec{a}| = 8|\vec{b}| = \frac{16\sqrt{2}}{3\sqrt{7}}$. **1**

∴ **|b| = 2√2/(3√7), |a| = 16√2/(3√7)**

24. Solution:

$2\vec{a} - \vec{b} + 3\vec{c} = (2 - 4 + 3)\hat{i} + (2 + 2 - 6)\hat{j} + (2 - 3 + 3)\hat{k} = \hat{i} - 2\hat{j} + 2\hat{k}$, with magnitude 3. **1**

Required vector = $6 \times \frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k}) = 2\hat{i} - 4\hat{j} + 4\hat{k}$. **1**

∴ **$2\hat{i} - 4\hat{j} + 4\hat{k}$**

25. Solution:

Z at the corner points: 0, 14, 27, 33 and 30 respectively. **1**

Maximum Z = 33 at (4, 5). **1**

∴ **Max Z = 33 at (4, 5)**

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SECTION C

26. Solution:

$|A| = -4 - 15 = -19 \neq 0$; adj $A = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$. **1**

$A^{-1} = (1/|A|)$ adj $A = \frac{1}{-19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19}A$. Hence shown. **1**

The system is AX = B with $B = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$: $X = A^{-1}B = \frac{1}{19}\begin{bmatrix} 16+3 \\ 40-2 \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 19 \\ 38 \end{bmatrix} \Rightarrow x = 1, y = 2$. **1**

∴ **x = 1, y = 2**

27. Solution:

$\frac{dx}{dt} = -5$ cm/min, $\frac{dy}{dt} = 4$ cm/min. **1**

(a) $P = 2(x + y) \Rightarrow \frac{dP}{dt} = 2(-5 + 4) = -2$ cm/min: the perimeter decreases at 2 cm/min. **1**

(b) $A = xy \Rightarrow \frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} = 8 \times 4 + 6 \times (-5) = 2$ cm²/min: the area increases at 2 cm²/min. **1**

∴ **(a) −2 cm/min (b) 2 cm²/min**

28. Solution:

$\frac{dx}{dt} = a\left(-\sin t + \frac{1}{\sin t}\right) = \frac{a\cos^2 t}{\sin t}$ and $\frac{dy}{dt}$ = a cos t, so $\frac{dy}{dx}$ = tan t. **1**

$\frac{d^2y}{dx^2}$ = sec²t × $\frac{dt}{dx}$ = sec²t × $\frac{\sin t}{a\cos^2 t} = \frac{\sin t}{a\cos^4 t}$. **1**

At t = $\frac{\pi}{3}$: $\frac{\sqrt{3}/2}{a/16} = \frac{8\sqrt{3}}{a}$. **1**

∴ **8√3/a**

OR

Put x = sin A, y = sin B: cos A + cos B = a(sin A − sin B) ⇒ 2 cos$\frac{A+B}{2}$ cos$\frac{A-B}{2}$ = 2a cos $\frac{A+B}{2}$ sin$\frac{A-B}{2}$. **1**

cot$\frac{A-B}{2}$ = a ⇒ A − B = 2 cot⁻¹a, i.e. sin⁻¹x − sin⁻¹y = 2 cot⁻¹a. **1**

Differentiating: $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. **1**

∴ **Proved**

29. Solution:

$\frac{dy}{dx}$ = (1 + x)(1 + y²) ⇒ $\frac{dy}{1+y^2}$ = (1 + x) dx. **1**

Integrating: tan⁻¹y = x + $\frac{x^2}{2}$ + C. **1**

x = 0, y = 0 ⇒ C = 0; the particular solution is y = tan$\left(x + \frac{x^2}{2}\right)$. **1**

∴ **y = tan(x + x²/2)**

OR

$\frac{dy}{dx}$ + sec²x · y = tan x sec²x; IF = e^(∫sec²x dx) = e^(tan x). **1**

y e^(tan x) = ∫ tan x sec²x e^(tan x) dx; put tan x = t: ∫ t e^t dt = (t − 1)e^t + C. **1**

y e^(tan x) = (tan x − 1)e^(tan x) + C ⇒ y = tan x − 1 + C e^(−tan x). **1**

∴ **y = tan x − 1 + Ce^(−tan x)**

30. Solution:

Any point of the line: M(3t + 6, 2t + 7, −2t + 7); PM = (3t + 5, 2t + 5, −2t + 4) for P(1, 2, 3). **1**

PM ⊥ line: 3(3t + 5) + 2(2t + 5) − 2(−2t + 4) = 17t + 17 = 0 ⇒ t = −1; foot M(3, 5, 9). **1**

M is the mid-point of P and its image Q(α, β, γ): α = 2(3) − 1 = 5, β = 2(5) − 2 = 8, γ = 2(9) − 3 = 15. Image (5, 8, 15). **1**

∴ **(5, 8, 15)**

OR

For the angle θ, cos θ ∝ a(b − c) + b(c − a) + c(a − b). **1**

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= ab − ac + bc − ab + ca − bc = 0. **1**

So cos θ = 0 and θ = 90°: the lines are perpendicular. **1**

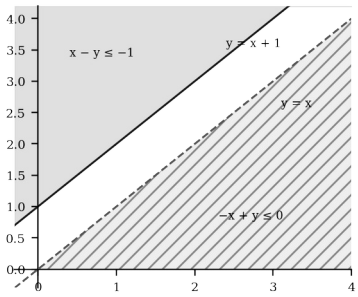
∴ **90°**

31. Solution:

$x - y \le -1$ is the region on or above the line $y = x + 1$; $-x + y \le 0$ is the region on or below the line $y = x$ (see figure). **1**

The lines $y = x + 1$ and $y = x$ are parallel, and no point can be both above $y = x + 1$ and below $y = x$. **1**

So there is no feasible region, and Z has no maximum value. **1**



∴ **No feasible region; no maximum**

SECTION D

32. Solution:

Reflexive: every polygon has the same number of sides as itself, so (P, P) ∈ R. **1**

Symmetric: if P_1 and P_2 have the same number of sides, so do P_2 and P_1; $(P_1, P_2) \in R \Rightarrow (P_2, P_1) \in R$. **1**

Transitive: if P_1, P_2 have the same number of sides and P_2, P_3 too, then P_1, P_3 have the same number of sides. So R is an equivalence relation. **1**

An element related to T must have the same number of sides as T, which is 3. **1**

So the set of all elements related to T is the set of all triangles in A. **1**

∴ **Equivalence relation; the set of all triangles**

OR

For $x \ge 0$, $f(x) = \frac{x}{1+x} \in [0, 1)$; for $x < 0$, $f(x) = \frac{x}{1-x} \in (-1, 0)$. So f(x) has the same sign as x. **1**

One-one: if $f(x_1) = f(x_2)$, x_1 and x_2 have the same sign. For $x_1, x_2 \ge 0$: $\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2} \Rightarrow x_1 = x_2$. **1**

For $x_1, x_2 < 0$: $\frac{x_1}{1-x_1} = \frac{x_2}{1-x_2} \Rightarrow x_1 = x_2$. So f is one-one. **1**

Onto: for $0 \le y < 1$, $x = \frac{y}{1-y} \ge 0$ gives f(x) = y; for $-1 < y < 0$, $x = \frac{y}{1+y} < 0$ gives $f(x) = \frac{x}{1-x} = y$. **1**

Every y in (−1, 1) has a pre-image, so f is onto. **1**

∴ **f is one-one and onto**

33. Solution:

Put sin x − cos x = t; then (cos x + sin x) dx = dt, and $t^2 = 1 - \sin 2x \Rightarrow \sin 2x = 1 - t^2$. **1**

$x = 0 \Rightarrow t = -1$; $x = \frac{\pi}{4} \Rightarrow t = 0$. $I = \int_{-1}^{0} \frac{dt}{9+16(1-t^2)} = \int_{-1}^{0} \frac{dt}{25-16t^2}$. **1**

$= \frac{1}{16}\int_{-1}^{0} \frac{dt}{\left(\frac{5}{4}\right)^2 - t^2} = \frac{1}{16} \times \frac{1}{2 \times \frac{5}{4}}\left[\log\left|\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right|\right]$ from −1 to 0. **1**

$= \frac{1}{40}\left[\log 1 - \log\frac{1}{9}\right] = \frac{1}{40}\log 9.$ **1**

$= \frac{1}{20}\log 3.$ **1**

∴ **(1/20) log 3**

34. Solution:

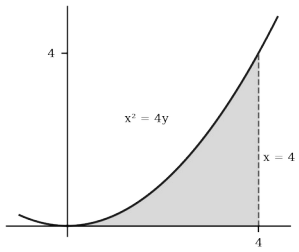
The curve is $y = \frac{x^2}{4}$, which is ≥ 0; the region lies between x = 0 and x = 4 above the x-axis (see figure). **1**

Area $= \int_0^4 \frac{x^2}{4}dx$. **1**

$= \frac{1}{4}\left[\frac{x^3}{3}\right]$ from 0 to 4. **1**

$= \frac{1}{4} \times \frac{64}{3}$. **1**

$= \frac{16}{3}$ square units. **1**



∴ **16/3 sq. units**

OR

For −1 ≤ y ≤ 1, x = 2y + 3 ≥ 1 > 0, so the line lies to the right of the y-axis (see figure). **1**

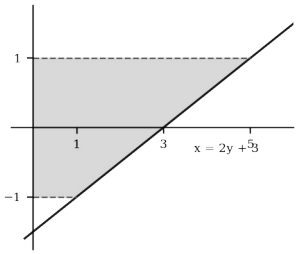
Area $= \int_{-1}^{1} (2y+3)dy$. **1**

$= [y^2 + 3y]$ from −1 to 1. **1**

$= (1 + 3) - (1 - 3) = 4 + 2.$ **1**

= 6 square units. **1**

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∴ **6 sq. units**

35. Solution:

Let E_1, E_2, E_3: seed of variety A_1, A_2, A_3; G: seed germinates. $P(E_1) = 0.4$, $P(E_2) = 0.4$, $P(E_3) = 0.2$; $P(G|E_1) = 0.45$, $P(G|E_2) = 0.60$, $P(G|E_3) = 0.35$. **1**

(a) By total probability, $P(G) = 0.4 \times 0.45 + 0.4 \times 0.60 + 0.2 \times 0.35 = 0.18 + 0.24 + 0.07 = 0.49$. **1**

(b) By Bayes' theorem, $P(E_2|G) = \frac{0.4 \times 0.60}{0.49} = \frac{0.24}{0.49}$. **1**

$= \frac{24}{49}$. **1**

(c) $P(G') = 1 - 0.49 = 0.51$. **1**

∴ **(a) 0.49 (b) 24/49 (c) 0.51**

SECTION E

36. Solution:

(i)

Direction $\overrightarrow{AB} = 2\hat{i} + 3\hat{j} + 6\hat{k}$; $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$. **1**

(ii)

$|\overrightarrow{AB}| = \sqrt{4+9+36} = 7$; direction cosines $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$. **1**

(iii)(a)

$\cos\theta = \frac{|2 \times 1 + 3 \times (-2) + 6 \times 2|}{7 \times 3} = \frac{8}{21}$. **1**

$\theta = \cos^{-1}\left(\frac{8}{21}\right)$. **1**

∴ **$\cos^{-1}$(8/21)**

OR

(iii)(b)

Point of L_1: $(2\lambda + 1, 3\lambda + 2, 6\lambda + 3)$; point of L_2: $(\mu + 4, -2\mu + 3, 2\mu + 11)$. Equating x and y: $2\lambda - \mu = 3$, $3\lambda + 2\mu = 1 \Rightarrow \lambda = 1, \mu = -1$. **1**

z: $6(1) + 3 = 9$ and $2(-1) + 11 = 9$ agree, so the paths intersect at (3, 5, 9), i.e. at B. **1**

∴ **(3, 5, 9)**

37. Solution:

(i)

$P'(t) = 18t - 3t^2$. **1**

(ii)

At 10 am, t = 3: $P'(3) = 54 - 27 = 27$ kW per hour (increasing). **1**

∴ **27 kW/h**

(iii)(a)

$P'(t) = 3t(6 - t) = 0 \Rightarrow t = 6$ (t = 0 is the start); $P''(t) = 18 - 6t = -18 < 0$ at t = 6. **1**

Maximum at t = 6, i.e. at 1 pm; maximum power $P(6) = 324 - 216 = 108$ kW. **1**

∴ **1 pm; 108 kW**

OR

(iii)(b)

$P'(t) = 3t(6 - t) > 0$ for $0 < t < 6$ and < 0 for $6 < t < 9$. **1**

Power increases from 7 am to 1 pm ($0 < t < 6$) and decreases from 1 pm to 4 pm ($6 < t < 9$). **1**

∴ **Increasing (0, 6); decreasing (6, 9)**

38. Solution:

(i)

$2x + y + z = 1500$, $x + 2y + z = 1700$, $x + y + 2z = 1600$; $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 1500 \\ 1700 \\ 1600 \end{bmatrix}$. **1**

$|A| = 2(4 - 1) - 1(2 - 1) + 1(1 - 2) = 6 - 1 - 1 = 4$. **1**

∴ **$|A| = 4$**

(ii)

$X = A^{-1}B = (1/|A|)(\text{adj } A)B = \frac{1}{4}\begin{bmatrix} 4500 - 1700 - 1600 \\ -1500 + 5100 - 1600 \\ -1500 - 1700 + 4800 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1200 \\ 2000 \\ 1600 \end{bmatrix}$. **1**

x = 300, y = 500, z = 400: football ₹300, basketball ₹500, volleyball ₹400. **1**

∴ **₹300, ₹500, ₹400**

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