MARKING SCHEME

MATHEMATICS (041)

Sample Paper 07 • Class XII • Session 2026–27

Q.P. Code : MC/SP/07 **Maximum Marks : 80**

Step-by-step solutions with the value points and marks for each step, as per the CBSE marking scheme. Marks for each step are shown in the right-hand column.

SECTION A

1. **Correct answer: (D) $\frac{\pi}{2}$** **1**

$\tan^{-1}(-1) = -\frac{\pi}{4}$ and $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

Sum $= -\frac{\pi}{4} + \frac{3\pi}{4} = \frac{\pi}{2}$.

2. **Correct answer: (C) $[0, \pi] - \{\frac{\pi}{2}\}$** **1**

sec is one-one on $[0, \pi] - \{\frac{\pi}{2}\}$ and takes every value in $\mathbb{R} - (-1, 1)$ there.

$\frac{\pi}{2}$ is excluded because sec $\frac{\pi}{2}$ is not defined.

3. **Correct answer: (B) $\frac{x}{\sqrt{1+x^2}}$** **1**

Let $\tan^{-1}x = \theta$; then $\tan\theta = x$ with $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so $\cos\theta > 0$ and $\sec\theta = \sqrt{1+x^2}$.

$\sin\theta = \tan\theta\cos\theta = \frac{x}{\sqrt{1+x^2}}$.

4. **Correct answer: (A) 4A** **1**

$A^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2A$.

$A^3 = A^2 \cdot A = 2A^2 = 4A$.

5. **Correct answer: (B) 2** **1**

$2x(x + 3) - 6x = 2x^2 = 8$.

$x^2 = 4 \Rightarrow x = 2$ ($x \in \mathbb{N}$).

6. **Correct answer: (A) 3 × 3** **1**

The product of an m × n matrix and an n × p matrix has order m × p.

Here m = 3, n = 1, p = 3, so the order is 3 × 3.

7. **Correct answer: (D) 6** **1**

Put $x = \frac{\pi}{2} + h$: $\cos x = -\sin h$ and $\pi - 2x = -2h$, so $f(x) = \frac{k\sin h}{2h} \to \frac{k}{2}$ as $h \to 0$.

Continuity: $\frac{k}{2} = 3 \Rightarrow k = 6$.

8. **Correct answer: (C) $\frac{1}{\sqrt{2}}$** **1**

Near $x = \frac{3\pi}{4}$, $\cos x < 0$, so $f(x) = -\cos x$ and $f'(x) = \sin x$.

$f'\left(\frac{3\pi}{4}\right) = \sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$.

9. **Correct answer: (A) 60π cm²/s** **1**

$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$.

$= 2\pi \times 10 \times 3 = 60\pi$ cm²/s.

10. **Correct answer: (B) cos x** **1**

$d/dx(\cos x) = -\sin x < 0$ on $\left(0, \frac{\pi}{2}\right)$, so cos x is decreasing there.

tan x increases; sin 2x and cos 3x change their behaviour inside the interval.

11. **Correct answer: (D) $\sin^{-1}y - \sin^{-1}x = C$** **1**

Separating: $\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$.

Integrating: $\sin^{-1}y = \sin^{-1}x + C$, i.e. $\sin^{-1}y - \sin^{-1}x = C$.

12. **Correct answer: (C) $\frac{\pi^2}{32}$** **1**

Put $\tan^{-1}x = t$, $\frac{dx}{1+x^2} = dt$; limits 0 to $\frac{\pi}{4}$.

$\int_0^{\frac{\pi}{4}} t\,dt = \frac{1}{2}\left(\frac{\pi}{4}\right)^2 = \frac{\pi^2}{32}$.

13. **Correct answer: (B) −7, 6** **1**

The vector is $(-5-2)\hat{i} + (7-1)\hat{j} = -7\hat{i} + 6\hat{j}$.

Its scalar components are −7 and 6.

14. **Correct answer: (D) $3\sqrt{3}$** **1**

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$.

$= 2 \times 3 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$.

15. **Correct answer: (C) $\sqrt{41}$** **1**

The foot of the perpendicular from (3, 4, 5) to the x-axis is (3, 0, 0).

Distance $= \sqrt{4^2+5^2} = \sqrt{41}$.

16. **Correct answer: (A) 1** **1**

$P(B|A) + P(B'|A) = \frac{P(A \cap B) + P(A \cap B')}{P(A)}$.

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$= \frac{P(A)}{P(A)} = 1.$

17. **Correct answer: (A) $\frac{2}{7}$** **1**

A leap year has 52 weeks and 2 extra days; the 2 extra days are one of 7 equally likely pairs (Sun–Mon, Mon–Tue, …, Sat–Sun).

Two of these pairs contain a Sunday, so P = $\frac{2}{7}$.

18. **Correct answer: (B) $\frac{7}{2}$** **1**

X takes the values 1, 2, …, 6, each with probability $\frac{1}{6}$.

Mean = $\frac{1+2+3+4+5+6}{6} = \frac{21}{6} = \frac{7}{2}$.

19. **Correct answer: (B)** **1**

Putting $1 + x^2 = t$, $2x\,dx = dt$: $\int \frac{dt}{t} = \log t + C = \log(1 + x^2) + C$. So A is true.

$1 + x^2 > 0$ for all real x, so R is true; but R does not explain how the integral is obtained.

20. **Correct answer: (B)** **1**

For $y = e^x$, $\frac{d^2y}{dx^2} = e^x = y$, so $\frac{d^2y}{dx^2} - y = 0$: A is true.

The highest order derivative is the second, so R is true; but the order does not explain why e^x is a solution.

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SECTION B

21. **Solution:**

$\frac{x-3}{(x-1)^3} = \frac{(x-1)-2}{(x-1)^3} = \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} = f(x) + f'(x)$ with $f(x) = \frac{1}{(x-1)^2}$. **1**

$\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C = \frac{e^x}{(x-1)^2} + C.$ **1**

∴ **$e^x/(x-1)^2 + C$**

OR

$\frac{\cos 2x}{\cos x + \sin x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \cos x - \sin x.$ **1**

$\int_0^{\frac{\pi}{2}} (\cos x - \sin x)\,dx$ = [sin x + cos x] from 0 to $\frac{\pi}{2}$ = 1 − 1 = 0. **1**

∴ **0**

22. **Solution:**

$x = \frac{\sin y}{\sin(a+y)}$; $\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y\cos(a+y)}{\sin^2(a+y)} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$. **1**

Hence $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$. **1**

∴ **Proved**

23. Solution:

(A) Three Dimensional Geometry

Direction ratios: 3 − 3, −2 + 2, 6 + 5 = 0, 0, 11. **1**

Cartesian equation: (x − 3)/0 = (y + 2)/0 = (z + 5)/11, i.e. the line x = 3, y = −2 (parallel to the z-axis). **1**

∴ **(x − 3)/0 = (y + 2)/0 = (z + 5)/11**

OR

(B) Vector Algebra

$\vec{a} \cdot \vec{b} = 3 + 4 + 3 = 10$; $|\vec{a}| = |\vec{b}| = \sqrt{14}$. **1**

$\cos\theta = \frac{10}{14} = \frac{5}{7} \Rightarrow \theta = \cos^{-1}\left(\frac{5}{7}\right)$. **1**

∴ **$\cos^{-1}(5/7)$**

24. Solution:

$\vec{a} + \vec{b} = -\vec{c} \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{c}|^2 \Rightarrow 9 + 25 + 2\vec{a} \cdot \vec{b} = 49 \Rightarrow \vec{a} \cdot \vec{b} = \frac{15}{2}$. **1**

$\cos\theta = \frac{15/2}{3 \times 5} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$. **1**

∴ **$\pi/3$**

25. Solution:

x + 2y = 10 and 3x + 4y = 24 meet at (4, 3); corner points: (0, 5), (4, 3), (0, 6). **1**

Z = 2500, 2300, 3000 respectively; minimum Z = 2300 at (4, 3). **1**

∴ **Min Z = 2300 at (4, 3)**

SECTION C

26. Solution:

X must be of order 2 × 2; let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix}$. **1**

Equating the first row: a + 4b = −7, 2a + 5b = −8 ⇒ a = 1, b = −2 (and 3a + 6b = −9 checks). **1**

Second row: c + 4d = 2, 2c + 5d = 4 ⇒ c = 2, d = 0 (and 3c + 6d = 6 checks). So $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$. **1**

∴ **X = [[1, −2], [2, 0]]**

27. Solution:

f′(x) = 16x^(1/3) − 2x^(−2/3) = $\frac{2(8x-1)}{x^{(2/3)}}$; f′(x) = 0 at x = $\frac{1}{8}$, and f′ is not defined at x = 0. **1**

f(−1) = 12 + 6 = 18, f(0) = 0, $f\left(\frac{1}{8}\right) = \frac{12}{16} - \frac{6}{2} = -\frac{9}{4}$, f(1) = 6. **1**

Absolute maximum = 18 at x = −1; absolute minimum = $-\frac{9}{4}$ at x = $\frac{1}{8}$. **1**

∴ **Max 18 at x = −1; Min −9/4 at x = 1/8**

28. Solution:

$\sqrt{1-x^2}$ y = $\sin^{-1}$x. Differentiating: $\sqrt{1-x^2}\frac{dy}{dx} - \frac{xy}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} \Rightarrow (1 - x^2)\frac{dy}{dx} - xy = 1.$ **1**

Differentiating again: $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - x\frac{dy}{dx} - y = 0.$ **1**

$(1 - x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$. Hence shown. **1**

∴ **Shown**

OR

$\frac{dx}{dt}$ = −2 sin 2t · e^(cos 2t) and $\frac{dy}{dt}$ = 2 cos 2t · e^(sin 2t). **1**

$\frac{dy}{dx} = -\frac{\cos 2t \cdot e^{(}\sin 2t)}{\sin 2t \cdot e^{(}\cos 2t)} = -\frac{y \cos 2t}{x \sin 2t}$. **1**

Since log x = cos 2t and log y = sin 2t, $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$. Hence proved. **1**

∴ **Proved**

29. Solution:

Linear with P = −3 cot x; IF = e^(−3 log sin x) = $\frac{1}{\sin^3 x}$. **1**

$\frac{y}{\sin^3 x} = \int \frac{2 \sin x \cos x}{\sin^3 x} dx = \int 2 \text{ cosec } x \cot x \, dx = -2 \text{ cosec } x + C \Rightarrow y = -2 \sin^2 x + C \sin^3 x.$ **1**

$x = \frac{\pi}{2}$, y = 2 ⇒ 2 = −2 + C ⇒ C = 4; $y = 4 \sin^3 x - 2 \sin^2 x$. **1**

∴ **$y = 4 \sin^3 x - 2 \sin^2 x$**

OR

$\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$; put y = vx: $v + x\frac{dv}{dx} = \frac{1 + v^2}{1 + v} \Rightarrow x\frac{dv}{dx} = \frac{1 - v}{1 + v}$. **1**

$\frac{1+v}{1-v} dv = \frac{dx}{x} \Rightarrow \int\left(-1 + \frac{2}{1-v}\right) dv = \log|x| + C \Rightarrow -v - 2\log|1 - v| = \log|x| + C.$ **1**

Putting $v = \frac{y}{x}$ and simplifying: $(x - y)^2 = Cx \, e^{(-y/x)}$. **1**

∴ **$(x - y)^2 = Cx \, e^{(-y/x)}$**

30. Solution:

Rewriting: $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + t(-\hat{i} + \hat{j} - 2\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$. **1**

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$, magnitude $\sqrt{29}$; $\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}$. **1**

SD = $\frac{|(2\hat{i}-4\hat{j}-3\hat{k})\cdot(\hat{j}-4\hat{k})|}{\sqrt{29}} = \frac{|-4+12|}{\sqrt{29}} = \frac{8}{\sqrt{29}}$ units. **1**

∴ **8/√29 units**

OR

Direction ratios: 5 − 4, k − 1, 0 − 2 = 1, k − 1, −2 and 3 − 2, 3 − 1, −1 − 1 = 1, 2, −2. **1**

Perpendicular: 1 × 1 + 2(k − 1) + (−2)(−2) = 0. **1**

$2k + 3 = 0 \Rightarrow k = -\frac{3}{2}$. **1**

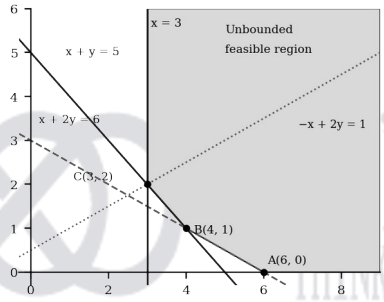
∴ **k = −3/2**

31. Solution:

Draw x = 3, x + y = 5 and x + 2y = 6. The feasible region is unbounded, with corner points A(6, 0), B(4, 1) and C(3, 2) (see figure). **1**

Z at the corners: A: −6; B: −2; C: 1. The largest of these is 1. **1**

As the region is unbounded, draw −x + 2y > 1: this half-plane has points in common with the feasible region. So Z has no maximum value. **1**



∴ **Z has no maximum value**

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SECTION D

32. Solution:

Reflexive: a − a = 0 is divisible by 2, so (a, a) ∈ R for all a ∈ Z. **1**

Symmetric: if 2 divides a − b, then 2 divides −(a − b) = b − a; so (a, b) ∈ R ⇒ (b, a) ∈ R. **1**

Transitive: if 2 divides a − b and b − c, then 2 divides (a − b) + (b − c) = a − c. So R is an equivalence relation. **1**

[0] = {b ∈ Z : (0, b) ∈ R} = {b ∈ Z : 2 divides 0 − b}. **1**

So [0] = {0, ±2, ±4, ±6, …}, the set of all even integers. **1**

∴ **Equivalence relation; [0] = set of even integers**

OR

$f(1) = \frac{2}{2} = 1$ and $f(2) = \frac{2}{2} = 1$. **1**

So f(1) = f(2) with 1 ≠ 2: f is not one-one. **1**

Onto: let m ∈ N. Take n = 2m (even); then f(n) = 2m/2 = m. **1**

Thus every m ∈ N has a pre-image, so f is onto. **1**

Since f is onto but not one-one, f is not bijective. **1**

∴ **Not bijective (onto, not one-one)**

33. Solution:

Let I be the integral. Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$: I = $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos x + \sin x}\,dx$. **1**

Adding: 2I = $\int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x}\,dx = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} cosec\left(x + \frac{\pi}{4}\right)dx$, since sin x + cos x = $\sqrt{2}$ sin $\left(x + \frac{\pi}{4}\right)$. **1**

= $\frac{1}{\sqrt{2}}\left[\log\left|\tan\left(\frac{x}{2} + \frac{\pi}{8}\right)\right|\right]$ from 0 to $\frac{\pi}{2}$ = $\frac{1}{\sqrt{2}}\left[\log\tan\frac{3\pi}{8} - \log\tan\frac{\pi}{8}\right]$. **1**

$\tan\frac{3\pi}{8} = \sqrt{2} + 1$ and $\tan\frac{\pi}{8} = \sqrt{2} - 1 = \frac{1}{\sqrt{2}+1}$, so 2I = $\frac{1}{\sqrt{2}} \times 2\log(\sqrt{2} + 1)$. **1**

I = $\frac{1}{\sqrt{2}}\log(\sqrt{2} + 1)$. **1**

∴ **$(1/\sqrt{2})\log(\sqrt{2} + 1)$**

34. Solution:

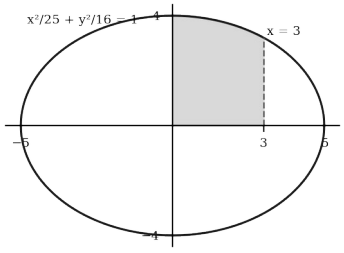
In the first quadrant, y = $\frac{4}{5}\sqrt{25 - x^2}$ (see figure). **1**

Area = $\frac{4}{5}\int_0^3 \sqrt{25 - x^2}\,dx$. **1**

= $\frac{4}{5}\left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]$ from 0 to 3. **1**

= $\frac{4}{5}\left[\frac{3}{2} \times 4 + \frac{25}{2}\sin^{-1}\frac{3}{5}\right]$. **1**

= $\frac{24}{5}$ + 10 $\sin^{-1}\frac{3}{5}$ square units. **1**



∴ **24/5 + 10 $\sin^{-1}(3/5)$ sq. units**

OR

sin x ≥ 0 on [0, π] and sin x ≤ 0 on [π, 2π] (see figure). **1**

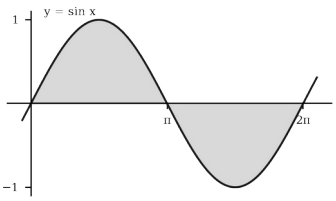
Area = $\int_0^{\pi} \sin x\,dx + \left|\int_{\pi}^{2\pi} \sin x\,dx\right|$. **1**

$\int_0^{\pi} \sin x\,dx$ = [−cos x] from 0 to π = 2. **1**

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$\int_{\pi}^{2\pi} \sin x\,dx$ = [−cos x] from π to 2π = −2, so its absolute value is 2. **1**

Area = 2 + 2 = 4 square units. **1**



∴ **4 sq. units**

35. Solution:

Let E_1: person has the disease, E_2: person is healthy, A: test is positive. $P(E_1) = 0.001$, $P(E_2) = 0.999$; $P(A|E_1) = 0.99$, $P(A|E_2) = 0.005$. **1**

(a) By total probability, $P(A) = 0.001 \times 0.99 + 0.999 \times 0.005 = 0.00099 + 0.004995 = 0.005985$. **1**

(b) By Bayes' theorem, $P(E_1|A) = \frac{0.00099}{0.005985} = \frac{990}{5985}$. **1**

$= \frac{22}{133}$. **1**

(c) $P(E_2|A) = 1 - \frac{22}{133} = \frac{111}{133}$. **1**

∴ **(a) 0.005985 (b) 22/133 (c) 111/133**

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SECTION E

36. Solution:

(i)

Direction ratios 2, −1, 2 with $\sqrt{4+1+4} = 3$; direction cosines $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$. **1**

(ii)

(7 + 1)/2 = 4 but (0 − 3)/(−1) = 3: the ratios are not equal, so the point does not lie on L. **1**

∴ **No**

(iii)(a)

Any point of L: F(2t − 1, 3 − t, 2t + 2); TF = (2t − 6, −t, 2t − 3). TF ⊥ L: 2(2t − 6) + t + 2(2t − 3) = 9t − 18 = 0 ⇒ t = 2. **1**

Nearest point F(3, 1, 6); TF = $\sqrt{4+4+1} = 3$ units (300 m). **1**

∴ **(3, 1, 6); 3 units**

OR

(iii)(b)

With the y-axis: $\cos\beta = -\frac{1}{3} \Rightarrow \beta = \cos^{-1}\left(-\frac{1}{3}\right) = \pi - \cos^{-1}\frac{1}{3}$. **1**

With the z-axis: $\cos\gamma = \frac{2}{3} \Rightarrow \gamma = \cos^{-1}\frac{2}{3}$. **1**

∴ **$\cos^{-1}(-1/3)$, $\cos^{-1}(2/3)$**

37. Solution:

(i)

By similar triangles, $\frac{r}{h} = \frac{5}{10} \Rightarrow r = \frac{h}{2}$; $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\frac{h^2}{4}h = \frac{\pi h^3}{12}$. **1**

(ii)

$\frac{dV}{dh} = \frac{\pi h^2}{4}$. **1**

(iii)(a)

$\frac{dV}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt} \Rightarrow 8\pi = \frac{\pi \times 16}{4}\frac{dh}{dt}$. **1**

$\frac{dh}{dt}$ = 2 m/h: the water level rises at 2 m per hour. **1**

∴ **2 m/h**

OR

(iii)(b)

$\frac{dV}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt}$ gives $\frac{dh}{dt}$ = 2 m/h at h = 4. **1**

$r = \frac{h}{2} \Rightarrow \frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$ = 1 m/h. **1**

∴ **1 m/h**

38. Solution:

(i)

$3x + 2y + z = 170$, $x + 2y + 3z = 190$, $2x + y + 2z = 160$; $A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 170 \\ 190 \\ 160 \end{bmatrix}$. **1**

$|A| = 3(4 - 3) - 2(2 - 6) + 1(1 - 4) = 3 + 8 - 3 = 8$. **1**

∴ **|A| = 8**

(ii)

$X = A^{-1}B = (1/|A|)(\text{adj } A)B = \frac{1}{8}\begin{bmatrix} 170 - 570 + 640 \\ 680 + 760 - 1280 \\ -510 + 190 + 640 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 240 \\ 160 \\ 320 \end{bmatrix}$. **1**

x = 30, y = 20, z = 40: sandwich ₹30, juice ₹20, cake ₹40. **1**

∴ **₹30, ₹20, ₹40**

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