# MARKING SCHEME

## MATHEMATICS (041)

**Sample Paper 08 • Class XII • Session 2026–27**

**Q.P. Code : MC/SP/08** **Maximum Marks : 80**

*Step-by-step solutions with the value points and marks for each step, as per the CBSE marking scheme. Marks for each step are shown in the right-hand column.*

## SECTION A

**1.** **Correct answer: (A) $-\frac{\pi}{3}$** **1**

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ and $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$.

Value $= \frac{\pi}{6} - \frac{\pi}{2} = -\frac{\pi}{3}$.

**2.** **Correct answer: (D) $\cos^{-1}2$** **1**

The domain of $\cos^{-1}$ is $[-1, 1]$, and 2 does not lie in it.

$\sin^{-1}\left(\frac{1}{2}\right)$, $\tan^{-1}2$ (domain R) and $\sec^{-1}2$ (domain $R - (-1, 1)$) are all defined.

**3.** **Correct answer: (C) 0.96** **1**

Let $\sin^{-1}(0.6) = \theta$; then $\sin\theta = 0.6$ and $\cos\theta = 0.8$ ($\theta$ acute).

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times 0.6 \times 0.8 = 0.96$.

**4.** **Correct answer: (B) O** **1**

$A^2 = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$.

So $A^2 = O$, although $A \neq O$.

**5.** **Correct answer: (D) 1** **1**

$\Delta = x \times x - (x + 1)(x - 1)$.

$= x^2 - (x^2 - 1) = 1$.

**6.** **Correct answer: (A) a symmetric matrix** **1**

$A' = -A$.

$(A^2)' = (A')^2 = (-A)^2 = A^2$, so $A^2$ is symmetric.

**7.** **Correct answer: (C) 2** **1**

[x] is discontinuous exactly at the integers.

The integers in (0, 3) are 1 and 2, so there are 2 points of discontinuity.

**8.** **Correct answer: (B) −e^(y − x)** **1**

Differentiating: $e^x + e^y\frac{dy}{dx} = e\text{^}(x + y)\left(1 + \frac{dy}{dx}\right) = (e^x + e^y)\left(1 + \frac{dy}{dx}\right)$.

$e^x + e^y\frac{dy}{dx} = e^x + e^y + (e^x + e^y)\frac{dy}{dx} \Rightarrow -e^x\frac{dy}{dx} = e^y \Rightarrow \frac{dy}{dx} = -e\text{^}(y - x)$.

**9.** **Correct answer: (A) 3.6 cm²/s** **1**

$V = x^3 \Rightarrow 3x^2\frac{dx}{dt} = 9 \Rightarrow \frac{dx}{dt} = \frac{9}{300} = 0.03$ cm/s at x = 10.

$S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x\frac{dx}{dt} = 12 \times 10 \times 0.03 = 3.6$ cm²/s.

**10.** **Correct answer: (D) |x| > 3** **1**

$f'(x) = 3x^2 - 27 = 3(x^2 - 9)$.

$f'(x) > 0 \Leftrightarrow x^2 > 9 \Leftrightarrow |x| > 3$.

**11.** **Correct answer: (B) $\frac{dy}{dx} = y$** **1**

Differentiating $y = a e^x$: $\frac{dy}{dx} = a e^x$.

Eliminating a using $y = a e^x$: $\frac{dy}{dx} = y$.

**12.** **Correct answer: (C) tan x + sec x + C** **1**

$\sec x (\sec x + \tan x) = \sec^2 x + \sec x \tan x$.

$\int(\sec^2 x + \sec x \tan x)\, dx = \tan x + \sec x + C$.

**13.** **Correct answer: (C) $\frac{2\pi}{3}$** **1**

$|\vec{a} + \vec{b}|^2 = 1 + 1 + 2\cos\theta = 1 \Rightarrow \cos\theta = -\frac{1}{2}$.

$\theta = \frac{2\pi}{3}$.

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**14.** **Correct answer: (B) 0** **1**

Projection = $\frac{(\hat{i} - \hat{j})\cdot(\hat{i} + \hat{j})}{\square} = \frac{1 - 1}{\sqrt{2}}$.

= 0 (the two vectors are perpendicular).

**15.** **Correct answer: (D) k = $\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$** **1**

For direction cosines, $l^2 + m^2 + n^2 = 1 \Rightarrow 3k^2 = 1$.

$k = \pm\frac{1}{\sqrt{3}}$.

**16.** **Correct answer: (A) $\frac{2}{3}$** **1**

$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.3 - 0.7 = 0.2$.

$P(A|B) = \frac{0.2}{0.3} = \frac{2}{3}$.

**17.** **Correct answer: (C) independent** **1**

$P(E) = \frac{13}{52} = \frac{1}{4}$, $P(F) = \frac{4}{52} = \frac{1}{13}$, $P(E \cap F) = \frac{1}{52}$.

P(E) P(F) = $\frac{1}{52}$ = P(E ∩ F), so E and F are independent.

**18. Correct answer: (A) 1.1** **1**

0.2 + 0.5 + k = 1 ⇒ k = 0.3.

Mean = 0 × 0.2 + 1 × 0.5 + 2 × 0.3 = 1.1.

**19. Correct answer: (D)** **1**

R is a standard integral — R is true.

With a = 3, R gives $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)$ + C, so the factor $\frac{1}{3}$ is missing in A. A is false.

**20. Correct answer: (A)** **1**

R is the definition of a first order linear differential equation — R is true.

The equation can be written $\frac{dy}{dx} - \frac{1}{x}y = 1$, with P = $-\frac{1}{x}$ and Q = 1. A is true and R explains it.

## SECTION B

**21. Solution:**

Put $x^3 = t$, $3x^2\,dx = dt$: $\int \frac{x^2}{1+x^6}\,dx = \frac{1}{3}\int \frac{dt}{1+t^2}$. **1**

$= \frac{1}{3}\tan^{-1}t + C = \frac{1}{3}\tan^{-1}(x^3) + C$. **1**

**$^{-}$(1/3) tan $^{1}$(x$^3$) + C**

**OR**

$1 - \sin 2x = (\sin x - \cos x)^2$, and for $\frac{\pi}{4} < x < \frac{\pi}{2}$, sin x > cos x, so $\sqrt{1-\sin 2x} = \sin x - \cos x$. **1**

$\int(\sin x - \cos x)\,dx = -\cos x - \sin x + C$. **1**

**∴ −cos x − sin x + C**

**22. Solution:**

$\frac{dy}{dx} = -5\sin x - 3\cos x$. **1**

$\frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -(5\cos x - 3\sin x) = -y$; hence $\frac{d^2y}{dx^2} + y = 0$. **1**

**∴ Proved**

**23. Solution:**

**(A) Three Dimensional Geometry**

Direction $-4\hat{i} + 2\hat{j} + 0\hat{k}$; $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-4\hat{i} + 2\hat{j})$. **1**

$(-4\hat{i} + 2\hat{j}) \cdot \hat{k} = 0$, so the line is perpendicular to the z-axis. **1**

**∴ $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-4\hat{i} + 2\hat{j})$**

**OR**

**(B) Vector Algebra**

External division in the ratio m : n = 2 : 1: $\vec{r} = \frac{2\vec{q} - \vec{p}}{2-1}$. **1**

$= 2(-\hat{i} + \hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} - \hat{k}) = -3\hat{i} + 3\hat{k}$. **1**

**∴ $-3\hat{i} + 3\hat{k}$**

**24. Solution:**

$\overrightarrow{AB} = \hat{i} - 3\hat{j} + \hat{k}$, $\overrightarrow{AC} = 3\hat{i} + 3\hat{j} - 4\hat{k}$; $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix} = 9\hat{i} + 7\hat{j} + 12\hat{k}$. **1**

Area $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{81+49+144} = \frac{\sqrt{274}}{2}$ square units. **1**

**∴ √274/2 sq. units**

**25. Solution:**

Corner points of the feasible region: (0, 0), (2, 0), (2, 2), (0, 6). **1**

Z = 0, 10, 14, 12 respectively; maximum Z = 14 at (2, 2). **1**

**∴ Max Z = 14 at (2, 2)**

## SECTION C

**26. Solution:**

$A'A = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix}$. **1**

$A'A = I \Rightarrow 2x^2 = 1, 6y^2 = 1, 3z^2 = 1$. **1**

$x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}$. **1**

**∴ x = ±1/√2, y = ±1/√6, z = ±1/√3**

**27. Solution:**

Let the perimeter be P (fixed) and the sides x and y: $2(x + y) = P \Rightarrow y = \frac{P}{2} - x$; area $A = x\left(\frac{P}{2} - x\right) = \frac{Px}{2} - x^2$. **1**

$\frac{dA}{dx} = \frac{P}{2} - 2x = 0 \Rightarrow x = \frac{P}{4}$; $\frac{d^2A}{dx^2} = -2 < 0$, so A is maximum. **1**

Then $y = \frac{P}{2} - \frac{P}{4} = \frac{P}{4} = x$, so the rectangle of maximum area is a square. **1**

**∴ Proved**

**28. Solution:**

$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1}\left(1 + \frac{x}{\sqrt{1+x^2}}\right) = \frac{ny}{\sqrt{1+x^2}} \Rightarrow \sqrt{1+x^2}\frac{dy}{dx} = ny$. **1**

Differentiating again: $\sqrt{1+x^2}\frac{d^2y}{dx^2} + \frac{x}{\sqrt{1+x^2}}\frac{dy}{dx} = n\frac{dy}{dx} = \frac{n^2y}{\sqrt{1+x^2}}$. **1**

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Multiplying by $\sqrt{1+x^2}$: $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - n^2y = 0$. Hence shown. **1**

**∴ Shown**

**OR**

Taking logarithms: $y \log \cos x = x \log \cos y$. **1**

Differentiating: $\frac{dy}{dx} \log \cos x - y \tan x = \log \cos y - x \tan y \frac{dy}{dx}$. **1**

$\frac{dy}{dx}(\log \cos x + x \tan y) = \log \cos y + y \tan x \Rightarrow \frac{dy}{dx} = \frac{\log \cos y + y \tan x}{\log \cos x + x \tan y}$. **1**

**∴ (log cos y + y tan x)/(log cos x + x tan y)**

**29. Solution:**

Dividing by x: $\frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$; IF = e^(log x + log sin x) = x sin x. **1**

$xy \sin x = \int x \sin x \, dx = -x \cos x + \sin x + C$ (integration by parts). **1**

$y = \frac{-x \cos x + \sin x + C}{x \sin x}$, i.e. $y = -\cot x + \frac{1}{x} + \frac{C}{x \sin x}$. **1**

**∴ xy sin x = −x cos x + sin x + C**

**OR**

$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$: linear in x; IF = e^($\tan^{-1}$y). **1**

x e^($\tan^{-1}$y) = $\int \frac{\tan^{-1} y \cdot e^{(\tan^{-1} y)}}{1+y^2}$ dy; put $\tan^{-1}y = t$: $\int t e^t \, dt = (t - 1)e^t + C$. **1**

$x = \tan^{-1}y - 1 + C e^{(-\tan^{-1}y)}$. **1**

**∴ $x = \tan^{-1}y - 1 + Ce^{(-\tan^{-1}y)}$**

**30. Solution:**

Any point of the line: Q(5t − 3, 2t + 1, 3t − 4); PQ = (5t − 3, 2t − 1, 3t − 7). **1**

PQ ⊥ line: 5(5t − 3) + 2(2t − 1) + 3(3t − 7) = 38t − 38 = 0 ⇒ t = 1; foot Q(2, 3, −1). **1**

Length PQ = $\sqrt{4+1+16} = \sqrt{21}$ units. **1**

**∴ Foot (2, 3, −1); length √21 units**

**OR**

General points: (4s + 5, 4s + 7, −5s − 3) and (7t + 8, t + 4, 3t + 5). **1**

Equating x and y: 4s − 7t = 3 and 4s − t = −3 ⇒ t = −1, s = −1. **1**

z: −5(−1) − 3 = 2 and 3(−1) + 5 = 2 agree, so the lines intersect, at (1, 3, 2). **1**

**∴ (1, 3, 2)**

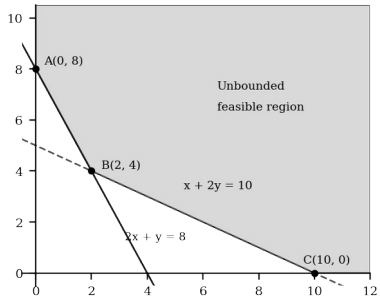
**31. Solution:**

Draw 2x + y = 8 and x + 2y = 10; they meet at B(2, 4). The feasible region is unbounded with corner points A(0, 8), B(2, 4), C(10, 0). **1**

Z at the corners: A: 560; B: 380; C: 500. **1**

The half-plane 50x + 70y < 380 has no point in common with the feasible region. Hence minimum Z = 380 at (2, 4). **1**

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**∴ Min Z = 380 at (2, 4)**

## SECTION D

**32. Solution:**

R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)}. **1**

Reflexive: every a divides itself, so (a, a) ∈ R for all a ∈ A. **1**

Transitive: if a divides b and b divides c, then a divides c; so (a, b), (b, c) ∈ R ⇒ (a, c) ∈ R. **1**

Not symmetric: (1, 2) ∈ R but (2, 1) ∉ R, since 1 is not divisible by 2. **1**

Hence R is reflexive and transitive but not symmetric. **1**

**∴ Shown**

**OR**

(i) f(1.2) = f(1.7) = 1 with 1.2 ≠ 1.7, so f is not one-one. **1**

The range of f is Z; a non-integer such as 0.5 has no pre-image, so f is not onto. **1**

(ii) g(−1) = g(1) = 1 with −1 ≠ 1, so g is not one-one. **1**

g(x) ≥ 0 for all x, so a negative number such as −1 has no pre-image; g is not onto. **1**

Hence both functions are neither one-one nor onto. **1**

**∴ Shown**

**33. Solution:**

By division, $x^4 = (x-1)(x^2+1)(x+1) + 1$, so the integrand $= x + 1 + \frac{1}{(x-1)(x^2+1)}$. **1**

Let $\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$; then $1 = A(x^2+1) + (Bx + C)(x-1)$. **1**

$x = 1 \Rightarrow A = \frac{1}{2}$; comparing $x^2$ terms: $A + B = 0 \Rightarrow B = -\frac{1}{2}$; constant terms: $A - C = 1 \Rightarrow C = -\frac{1}{2}$. **1**

$\int = \frac{x^2}{2} + x + \frac{1}{2}\int\frac{dx}{x-1} - \frac{1}{2}\int\frac{x+1}{x^2+1}\,dx$. **1**

$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C$. **1**

**∴ $x^2/2 + x + (1/2)\log|x-1| - (1/4)\log(x^2+1) - (1/2)\tan^{-1}x + C$**

**34. Solution:**

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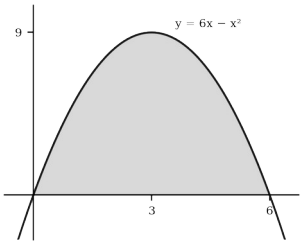
$y = x(6 - x) = 0 \Rightarrow x = 0, 6$; the curve is a downward parabola above the x-axis for $0 < x < 6$ (see figure). **1**

Area $= \int_0^6 (6x - x^2)\,dx$. **1**

$= \left[3x^2 - \frac{x^3}{3}\right]$ from 0 to 6. **1**

$= 108 - 72$. **1**

$= 36$ square units. **1**




**∴ 36 sq. units**

**OR**

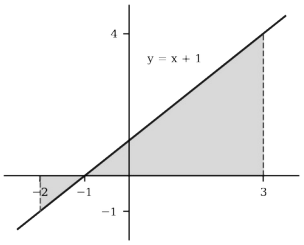
$y = x + 1 < 0$ for $-2 \le x < -1$ and $\ge 0$ for $-1 \le x \le 3$ (see figure). **1**

Area $= \left|\int_{-2}^{-1} (x+1)\,dx\right| + \int_{-1}^{3} (x+1)\,dx$. **1**

$\int_{-2}^{-1} (x+1)\,dx = \left[\frac{(x+1)^2}{2}\right]$ from $-2$ to $-1 = -\frac{1}{2}$, so this part has area $\frac{1}{2}$. **1**

$\int_{-1}^{3} (x+1)\,dx = \left[\frac{(x+1)^2}{2}\right]$ from $-1$ to $3 = 8$. **1**

Area $= \frac{1}{2} + 8 = \frac{17}{2}$ square units. **1**




**∴ 17/2 sq. units**

**35. Solution:**

X can take the values 0, 1, 2, 3; p = 0.2 (turbulence), q = 0.8 (no turbulence) for each flight. **1**

(a) $P(X = 0) = 0.8^3 = 0.512$; $P(X = 1) = 3 \times 0.2 \times 0.8^2 = 0.384$. **1**

$P(X = 2) = 3 \times 0.2^2 \times 0.8 = 0.096$; $P(X = 3) = 0.2^3 = 0.008$ (total = 1). **1**

(b) Mean $= 0 \times 0.512 + 1 \times 0.384 + 2 \times 0.096 + 3 \times 0.008 = 0.384 + 0.192 + 0.024 = 0.6$. **1**

(c) $P(X \ge 1) = 1 - P(X = 0) = 1 - 0.512 = 0.488$. **1**

**∴ (a) 0.512, 0.384, 0.096, 0.008 (b) 0.6 (c) 0.488**

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## SECTION E

**36. Solution:**

**(i)**

Direction ratios 1, −1, 0; direction cosines $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0$. **1**

**(ii)**

$(\hat{i} - \hat{j}) \cdot (\hat{j} - \hat{k}) = 0 - 1 + 0 = -1 \neq 0$, so they are not perpendicular. **1**

**∴ No**

**(iii)(a)**

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$, magnitude $\sqrt{3}$; $\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} + 3\hat{k}$. **1**

SD $= \frac{|2+1+3|}{\sqrt{3}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ units. **1**

**∴ 2√3 units**

**OR**

**(iii)(b)**

$\cos\theta = \frac{|-1|}{\sqrt{2} \times \sqrt{2}} = \frac{1}{2}$. **1**

$\theta = \frac{\pi}{3}$ (60°). **1**

**∴ π/3**

**37. Solution:**

**(i)**

$h'(t) = 12t - t^2$. **1**

**(ii)**

$h'(3) = 36 - 9 = 27$ cm per week. **1**

**∴ 27 cm/week**

**(iii)(a)**

Let $G(t) = 12t - t^2$; $G'(t) = 12 - 2t = 0 \Rightarrow t = 6$, and $G''(t) = -2 < 0$. **1**

The rate of growth is maximum after 6 weeks; maximum rate $G(6) = 72 - 36 = 36$ cm per week. **1**

**∴ 6 weeks; 36 cm/week**

**OR**

**(iii)(b)**

$h'(t) = t(12 - t) > 0$ for $0 < t < 12$, so the height keeps increasing. **1**

$h(12) = 6 \times 144 - 1728/3 + 5 = 864 - 576 + 5 = 293$ cm. **1**

**∴ 293 cm**

**38. Solution:**

**(i)**

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$x + y + z = 600$, $2x + y + 3z = 1300$, $x + 2y + z = 800$; $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 600 \\ 1300 \\ 800 \end{bmatrix}$. **1**

$|A| = 1(1 - 6) - 1(2 - 3) + 1(4 - 1) = -5 + 1 + 3 = -1$. **1**

**∴ $|A| = -1$**

**(ii)**

$X = A^{-1}B = (1/|A|)(\text{adj } A)B = -\begin{bmatrix} -3000+1300+1600 \\ 600+0-800 \\ 1800-1300-800 \end{bmatrix} = -\begin{bmatrix} -100 \\ -200 \\ -300 \end{bmatrix}$. **1**

$x = 100$, $y = 200$, $z = 300$: A costs ₹100, B ₹200 and C ₹300 per bag. **1**

**∴ ₹100, ₹200, ₹300**

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