MARKING SCHEME

MATHEMATICS (041)

Sample Paper 09 • Class XII • Session 2026–27

Q.P. Code : MC/SP/09 **Maximum Marks : 80**

Step-by-step solutions with the value points and marks for each step, as per the CBSE marking scheme. Marks for each step are shown in the right-hand column.

SECTION A

1. **Correct answer: (D) $\frac{5\pi}{6}$** **1**

$\cos^{-1}(-x) = \pi - \cos^{-1}x$ for $x \in [-1, 1]$, and $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$.

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$, which lies in $[0, \pi]$.

2. **Correct answer: (A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$** **1**

tan is one-one on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and takes every real value there.

The end points are excluded since $\tan\left(\pm\frac{\pi}{2}\right)$ is not defined.

3. **Correct answer: (B) $\frac{4}{5}$** **1**

Let $\tan^{-1}\frac{3}{4} = \theta$, so $\tan\theta = \frac{3}{4}$ with $\theta \in \left(0, \frac{\pi}{2}\right)$.

Then $\sec\theta = \sqrt{1+\frac{9}{16}} = \frac{5}{4}$, so $\cos\theta = \frac{4}{5}$.

4. **Correct answer: (C) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$** **1**

$Q = \frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}\right)$.

$= \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$.

5. **Correct answer: (B) $\frac{15}{2}$ sq. units** **1**

Area $= \frac{1}{2}\left|\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}\right| = \frac{1}{2}|1(0-3) - 0 + 1(18-0)|$.

$= \frac{1}{2} \times 15 = \frac{15}{2}$ sq. units.

6. **Correct answer: (C) 3I** **1**

$A^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$.

$A^2 - 2A = \begin{bmatrix} 5-2 & 4-4 \\ 4-4 & 5-2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 3I$.

7. **Correct answer: (D) continuous everywhere but not differentiable at x = 0** **1**

e^u and |x| are continuous, so their composite is continuous everywhere.

At x = 0, LHD = −1 and RHD = 1 (from e^(−x) and e^x), so f is not differentiable at x = 0.

8. **Correct answer: (A)** $\frac{1}{x \log x}$ **1**

By the chain rule, $\frac{dy}{dx} = \frac{1}{\log x} \times \frac{1}{x}$.

$= \frac{1}{x \log x}$.

9. **Correct answer: (A) 3 cm** **1**

$A = \pi r^2$, $C = 2\pi r$; $\frac{dA}{dC} = \frac{dA/dr}{dC/dr} = \frac{2\pi r}{2\pi} = r$.

At r = 3 cm, dA/dC = 3 cm.

10. **Correct answer: (C) 89** **1**

$f'(x) = 6x^2 - 24 = 0 \Rightarrow x = 2$ in [1, 3].

f(1) = 85, f(2) = 75, f(3) = 89; the absolute maximum is 89.

11. **Correct answer: (B) $y'y'' + y = \sin x$** **1**

The order is the order of the highest derivative present.

Only (b) has y'' as its highest derivative; (a) and (d) are of order 1 and (c) of order 3.

12. **Correct answer: (B) 2 sin $\sqrt{x}$ + C** **1**

Put $\sqrt{x} = t$, $\frac{dx}{2\sqrt{x}} = dt$.

$\int 2 \cos t \, dt = 2 \sin t + C = 2 \sin \sqrt{x} + C$.

13. **Correct answer: (D) [0, 12]** **1**

$|\lambda \vec{a}| = |\lambda| \, |\vec{a}| = 4|\lambda|$.

For $-3 \le \lambda \le 2$, $|\lambda|$ ranges over [0, 3], so $|\lambda \vec{a}|$ ranges over [0, 12].

14. **Correct answer: (C)** $\frac{\sqrt{229}}{2}$ **1**

$\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = -9\hat{i} + 2\hat{j} + 12\hat{k}$, with magnitude $\sqrt{229}$.

Area = $\frac{1}{2}\sqrt{229}$.

15. **Correct answer: (B) 3, 5, 3** **1**

Direction ratios: 2 − (−1), 3 − (−2), 4 − 1.

= 3, 5, 3.

16. **Correct answer: (A) 0.35** **1**

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)}$ (independence).

$= P(A) = 0.35$.

17. **Correct answer: (D) $\frac{15}{32}$** **1**

With replacement the draws are independent: $P(RB) = P(BR) = \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$.

Required probability $= 2 \times \frac{15}{64} = \frac{15}{32}$.

18. **Correct answer: (D) 0** **1**

Mean $= (-1)(0.3) + 0(0.4) + 1(0.3)$.

$= -0.3 + 0 + 0.3 = 0$.

19. **Correct answer: (B)** **1**

$\frac{d}{dx}(\log|\sec x|) = \frac{1}{\sec x} \times \sec x \tan x = \tan x$, so A is true.

$\tan(-x) = -\tan x$, so R is true; but the oddness of tan x does not explain the value of the indefinite integral.

20. **Correct answer: (C)** **1**

The highest order derivative is $\frac{d^2y}{dx^2}$, and its highest power is 3; the equation is a polynomial in the derivatives, so the degree is 3. A is true.

The degree is the power of the highest order derivative, not of the lowest order one, so R is false.

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SECTION B

21. **Solution:**

$\frac{1}{\sin x \cos x} = \frac{\sec^2 x}{\tan x}$, so the integral = $[\log|\tan x|]$ from $\frac{\pi}{6}$ to $\frac{\pi}{3}$. **1**

$= \log\sqrt{3} - \log\frac{1}{\sqrt{3}} = \log 3$. **1**

∴ **log 3**

OR

By parts, taking x as the first function: $\int x \sec^2 x\, dx = x \tan x - \int \tan x\, dx$. **1**

$= x \tan x - \log|\sec x| + C$. **1**

∴ **$x \tan x - \log|\sec x| + C$**

22. **Solution:**

$xy = \sqrt{a(\sin^{-1}t + \cos^{-1}t)}$. Since $\sin^{-1}t + \cos^{-1}t = \frac{\pi}{2}$, $xy = \sqrt{a(\pi/2)}$, a constant. **1**

Differentiating: $x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$. **1**

∴ **Shown**

23. Solution:

(A) Three Dimensional Geometry

Direction ratios: 3 − 1, 4 + 1, −2 − 2 = 2, 5, −4 and 3 − 0, 5 − 3, 6 − 2 = 3, 2, 4. **1**

2 × 3 + 5 × 2 + (−4) × 4 = 6 + 10 − 16 = 0, so the lines are perpendicular. **1**

∴ **Perpendicular**

OR

(B) Vector Algebra

$\vec{a} \cdot \vec{b} = |\vec{a}|^2 \cos 60° = \frac{1}{2}|\vec{a}|^2 = \frac{9}{2}$. **1**

$|\vec{a}|^2 = 9 \Rightarrow |\vec{a}| = |\vec{b}| = 3$. **1**

∴ **3 each**

24. Solution:

$\vec{d}$ is parallel to $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix} = 32\hat{i} - \hat{j} - 14\hat{k}$; let $\vec{d} = \lambda(32\hat{i} - \hat{j} - 14\hat{k})$. **1**

$\vec{c} \cdot \vec{d} = \lambda(64 + 1 - 56) = 9\lambda = 15 \Rightarrow \lambda = \frac{5}{3}$; $\vec{d} = \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$. **1**

∴ **(1/3)(160î − 5ĵ − 70k̂)**

25. Solution:

Corner points of the feasible region: (0, 0), (4, 0), (4, 2), (0, 6). **1**

Z = 0, 12, 16, 12 respectively; maximum Z = 16 at (4, 2). **1**

∴ **Max Z = 16 at (4, 2)**

SECTION C

26. Solution:

$A^2 = A \cdot A = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$. **1**

$A^3 = A^2 \cdot A = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$. **1**

$A^3 - 23A - 40I = \begin{bmatrix} 63-23-40 & 46-46 & 69-69 \\ 69-69 & -6+46-40 & 23-23 \\ 92-92 & 46-46 & 63-23-40 \end{bmatrix} = O$. Hence shown. **1**

∴ **Shown**

27. Solution:

Let $P = xy^3$ with $x = 60 - y$: $P(y) = (60 - y)y^3 = 60y^3 - y^4$. **1**

$P'(y) = 180y^2 - 4y^3 = 4y^2(45 - y) = 0 \Rightarrow y = 45$ ($y > 0$); $P''(y) = 360y - 12y^2 = -8100 < 0$ at $y = 45$. **1**

So P is maximum when y = 45 and x = 60 − 45 = 15. **1**

∴ **x = 15, y = 45**

28. Solution:

Let u = x^x: log u = x log x ⇒ du/dx = x^x(1 + log x). **1**

Let v = 2^(sin x): dv/dx = 2^(sin x) log 2 · cos x. **1**

$\frac{dy}{dx}$ = x^x(1 + log x) − 2^(sin x) cos x log 2. **1**

∴ **x^x(1 + log x) − 2^(sin x) cos x log 2**

OR

$\frac{dx}{dt}$ = a[2 cos 2t(1 + cos 2t) − 2 $\sin^2 2t$] = 2a(cos 2t + cos 4t). **1**

$\frac{dy}{dt}$ = b[−2 sin 2t(1 − cos 2t) + 2 sin 2t cos 2t] = 2b(−sin 2t + sin 4t). **1**

At t = $\frac{\pi}{4}$: $\frac{dx}{dt}$ = 2a(0 − 1) = −2a, $\frac{dy}{dt}$ = 2b(−1 + 0) = −2b; $\frac{dy}{dx} = \frac{b}{a}$. **1**

∴ **b/a**

29. Solution:

$\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x^2}$; IF = e^(∫dx/(x log x)) = e^(log(log x)) = log x. **1**

$y \log x = \int \frac{2}{x^2} \log x \, dx = 2\left[-\frac{\log x}{x} + \int \frac{1}{x^2} dx\right] = -\frac{2}{x}(1 + \log x) + C$. **1**

$y \log x = -\frac{2}{x}(1 + \log x) + C$ is the general solution. **1**

∴ **y log x = −(2/x)(1 + log x) + C**

OR

$\frac{dy}{dx}$ = log(x + 1) ⇒ dy = log(x + 1) dx. **1**

y = ∫ log(x + 1) dx = (x + 1) log(x + 1) − x + C (by parts). **1**

x = 0, y = 3 ⇒ C = 3; y = (x + 1) log(x + 1) − x + 3. **1**

∴ **y = (x + 1) log(x + 1) − x + 3**

30. Solution:

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$ = −9î + 3ĵ + 9k̂, magnitude $\sqrt{171} = 3\sqrt{19}$. **1**

$\vec{a}_2 - \vec{a}_1$ = 3î + 3ĵ + 3k̂; $(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)$ = −27 + 9 + 27 = 9. **1**

SD = $\frac{9}{3\sqrt{19}} = \frac{3}{\sqrt{19}}$ units. **1**

∴ **3/√19 units**

OR

Line AB: (4s, −1 + 6s, −1 + 2s); line CD: (3 − 7t, 9 − 5t, 4). **1**

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Equating z: $-1 + 2s = 4 \Rightarrow s = 5/2$, giving the point (10, 14, 4) on AB; equating x: $3 - 7t = 10 \Rightarrow t = -1$. **1**

y on CD at $t = -1$: $9 + 5 = 14$ ✓. So the lines intersect at (10, 14, 4). **1**

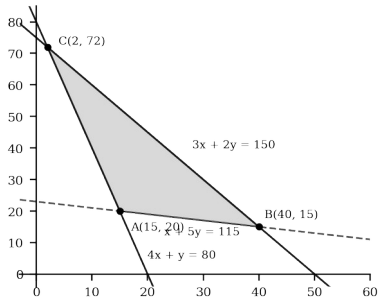
∴ **(10, 14, 4)**

31. Solution:

The lines meet at A(15, 20), B(40, 15) and C(2, 72); the feasible region is the triangle ABC (see figure). **1**

Z at the corners: A: 150; B: 285; C: 228. **1**

Minimum Z = 150 at x = 15, y = 20. **1**



∴ **Min Z = 150 at (15, 20)**

SECTION D

32. Solution:

Reflexive: $a^2 - 4a \cdot a + 3a^2 = 0$ for every real a, so $(a, a) \in R$: R is reflexive. **1**

Symmetric: $(3, 1) \in R$ since $9 - 12 + 3 = 0$; but $(1, 3) \notin R$ since $1 - 12 + 27 = 16 \neq 0$. **1**

So R is not symmetric. **1**

Transitive: $(9, 3) \in R$ ($81 - 108 + 27 = 0$) and $(3, 1) \in R$; but $(9, 1) \notin R$, since $81 - 36 + 3 = 48 \neq 0$. **1**

So R is not transitive. R is reflexive only. **1**

∴ **Reflexive; neither symmetric nor transitive**

OR

$f(x_1) = f(x_2) \Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4} \Rightarrow 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$. **1**

$\Rightarrow x_1 = x_2$, so f is one-one. **1**

For the range, let $y = \frac{4x}{3x+4}$: $3xy + 4y = 4x \Rightarrow x = \frac{4y}{4-3y}$, which is defined for $y \neq \frac{4}{3}$. **1**

For $y \neq \frac{4}{3}$, $x = \frac{4y}{4-3y}$ is a real number different from $-\frac{4}{3}$ $\left(since\ \frac{4y}{4-3y} = -\frac{4}{3} \Rightarrow 12y = -16 + 12y, impossible\right)$, and f(x) = y. **1**

$y = \frac{4}{3}$ is never attained, so the range of f is $R - \{\frac{4}{3}\}$. **1**

∴ **f is one-one; range = R − {4/3}**

33. Solution:

$\sin x + \sin 2x = \sin x(1 + 2\cos x)$; $\int \frac{dx}{\sin x(1+2\cos x)} = \int \frac{\sin x\,dx}{(1-\cos^2 x)(1+2\cos x)}$. **1**

Put $\cos x = t$, $-\sin x\,dx = dt$: $I = -\int \frac{dt}{(1-t)(1+t)(1+2t)}$. **1**

$\frac{1}{(1-t)(1+t)(1+2t)} = \frac{1/6}{1-t} - \frac{1/2}{1+t} + \frac{4/3}{1+2t}$ $\left(put\ t = 1, -1, -\frac{1}{2}\right)$. **1**

$I = -\left[-\frac{1}{6}\log|1-t| - \frac{1}{2}\log|1+t| + \frac{2}{3}\log|1+2t|\right] + C$. **1**

$= \frac{1}{6}\log|1 - \cos x| + \frac{1}{2}\log|1 + \cos x| - \frac{2}{3}\log|1 + 2\cos x| + C$. **1**

∴ **(1/6)log|1 − cos x| + (1/2)log|1 + cos x| − (2/3)log|1 + 2cos x| + C**

34. Solution:

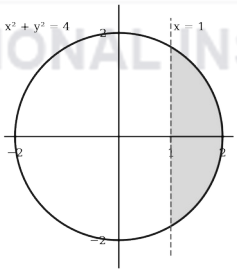
The region is the part of the disc $x^2 + y^2 \le 4$ to the right of the line $x = 1$; it is symmetric about the x-axis (see figure). **1**

Area $= 2\int_1^2 \sqrt{4 - x^2}\,dx$. **1**

$= 2\left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right]$ from 1 to 2. **1**

$= 2\left[(0+\pi) - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)\right]$. **1**

$= \frac{4\pi}{3} - \sqrt{3}$ square units. **1**



∴ **(4π/3 − √3) sq. units**

OR

The region lies between the y-axis and the curve $x = y^2$ for $1 \le y \le 3$ (see figure). **1**

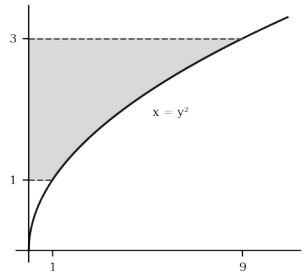
Area $= \int_1^3 y^2\,dy$. **1**

$= \left[\frac{y^3}{3}\right]$ from 1 to 3. **1**

$= 9 - \frac{1}{3}$. **1**

$= \frac{26}{3}$ square units. **1**

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∴ **26/3 sq. units**

35. Solution:

Let E: electricity bill paid on time, W: water bill paid on time; P(E) = 0.8, P(W) = 0.7, independent. (a) $P(E \cap W) = 0.8 \times 0.7 = 0.56$. **1**

(b) P(exactly one) = P(E) P(W′) + P(E′) P(W) = 0.8 × 0.3 + 0.2 × 0.7. **1**

= 0.24 + 0.14 = 0.38. **1**

(c) P(at least one) = 1 − P(E′) P(W′) = 1 − 0.2 × 0.3 = 0.94. **1**

(d) P(both | at least one) = $\frac{0.56}{0.94} = \frac{28}{47}$. **1**

∴ **(a) 0.56 (b) 0.38 (c) 0.94 (d) 28/47**

SECTION E

36. Solution:

(i)

Direction ratios 3 − 1, 2 − 1, 0 − 2 = 2, 1, −2; the beam: (x − 1)/2 = (y − 1)/1 = (z − 2)/(−2). **1**

(ii)

$\sqrt{4+1+4} = 3$; direction cosines $\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$. **1**

(iii)(a)

Any point of the beam: F(2t + 1, t + 1, 2 − 2t); PF = (2t − 4, t − 1, −2t). PF ⊥ beam: 2(2t − 4) + (t − 1) + 4t = 9t − 9 = 0 ⇒ t = 1. **1**

Foot F(3, 2, 0) (the point B); distance PF = $\sqrt{4+0+4} = 2\sqrt{2}$ m. **1**

∴ **(3, 2, 0); 2√2 m**

OR

(iii)(b)

The foot of the perpendicular from P is F(3, 2, 0) (from PF ⊥ beam with t = 1), and F is the mid-point of P and its image P′. **1**

P′ = 2F − P = (6 − 5, 4 − 2, 0 − 2) = (1, 2, −2). **1**

∴ **(1, 2, −2)**

37. Solution:

(i)

$x^2 + y^2 = 25$ (Pythagoras' theorem). **1**

(ii)

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$. **1**

(iii)(a)

When x = 4, y = $\sqrt{(25 - 16)} = 3$. **1**

$\frac{dy}{dt} = -\frac{4}{3} \times 0.5 = -\frac{2}{3}$ m/s: the top slides down at $\frac{2}{3}$ m/s. **1**

$\therefore$ **2/3 m/s**

OR

(iii)(b)

$x = 5 \cos\theta \Rightarrow \frac{dx}{dt} = -5 \sin\theta \frac{d\theta}{dt}$; when x = 4, y = 3 and $\sin\theta = \frac{3}{5}$. **1**

$0.5 = -5 \times \frac{3}{5} \times \frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = -\frac{1}{6}$ rad/s: θ decreases at $\frac{1}{6}$ radian per second. **1**

$\therefore$ **−1/6 rad/s**

38. Solution:

(i)

$x + y + z = 240$, $x - y = 10$, $x - 2y + z = 30$; $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 240 \\ 10 \\ 30 \end{bmatrix}$. **1**

$|A| = 1(-1 - 0) - 1(1 - 0) + 1(-2 + 1) = -1 - 1 - 1 = -3$. **1**

$\therefore$ **$|A| = -3$**

(ii)

$X = A^{-1}B = (1/|A|)(\text{adj } A)B = \frac{1}{-3}\begin{bmatrix} -240 - 30 + 30 \\ -240 + 0 + 30 \\ -240 + 30 - 60 \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -240 \\ -210 \\ -270 \end{bmatrix}$. **1**

$x = 80$, $y = 70$, $z = 90$: Mathematics 80, Physics 70, Chemistry 90. **1**

$\therefore$ **80, 70, 90**

• • • End of Marking Scheme • • •

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