MARKING SCHEME

MATHEMATICS (041)

Sample Paper 10 • Class XII • Session 2026–27

Q.P. Code : MC/SP/10 **Maximum Marks : 80**

Step-by-step solutions with the value points and marks for each step, as per the CBSE marking scheme. Marks for each step are shown in the right-hand column.

SECTION A

1. **Correct answer: (B) $-\frac{\pi}{6}$** **1**

$\tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$.

$-\frac{\pi}{6}$ lies in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, the principal value branch of $\tan^{-1}$, so the value is $-\frac{\pi}{6}$.

2. **Correct answer: (A) $\frac{\sqrt{4+y^2}}{2}$** **1**

Let $\tan^{-1}\frac{y}{2} = \theta$; then $\tan\theta = \frac{y}{2}$ with $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so $\sec\theta > 0$.

$\sec\theta = \sqrt{1+\frac{y^2}{4}} = \frac{\sqrt{4+y^2}}{2}$.

3. **Correct answer: (C) $\frac{\pi}{2}$** **1**

$\cos\frac{3\pi}{2} = 0$.

$\cos^{-1}0 = \frac{\pi}{2}$, which lies in $[0, \pi]$. $\left(Note: \frac{3\pi}{2}\ itself\ is\ not \in the\ principal\ branch.\right)$

4. **Correct answer: (D) B** **1**

$B^2 = B \cdot B = (BA)B = B(AB)$.

$= BA = B$.

5. **Correct answer: (A) 27 times the original** **1**

Multiplying one row by 3 multiplies the determinant by 3.

All three rows are multiplied by 3, so the value becomes $3^3 = 27$ times.

6. **Correct answer: (D) 8** **1**

In a diagonal matrix, all the non-diagonal entries are 0; only the 3 diagonal entries can be chosen.

Each diagonal entry has 2 choices, so there are $2^3 = 8$ such matrices.

7. **Correct answer: (B) no such value of k** **1**

As $x \to 0$, $\frac{1}{x} \to \pm\infty$ and $\sin\frac{1}{x}$ oscillates between -1 and 1, so $\lim(x \to 0)\ f(x)$ does not exist.

Hence f cannot be made continuous at 0 for any value of k.

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8. **Correct answer: (C) $\frac{2}{3x}$** **1**

Let u = x^2, v = x^3; $\frac{du}{dx} = 2x$, $\frac{dv}{dx} = 3x^2$.

$\frac{du}{dv} = \frac{2x}{3x^2} = \frac{2}{3x}$.

9. **Correct answer: (C) 1 m/h** **1**

$V = \pi r^2 h = 100\pi h \Rightarrow \frac{dV}{dt} = 100\pi \frac{dh}{dt}$.

$\frac{dh}{dt} = \frac{314}{314} = 1$ m/h.

10. **Correct answer: (B) a critical point which is neither a local maximum nor a local minimum** **1**

$f'(x) = 3x^2 = 0$ at x = 0, so 0 is a critical point.

$f'(x) > 0$ on both sides of 0 (no change of sign), so it is neither a local maximum nor a local minimum (a point of inflection).

11. **Correct answer: (A) $y = 5e^{-x}$** **1**

$\frac{dy}{y} = -dx \Rightarrow \log|y| = -x + C \Rightarrow y = A\,e^{-x}$.

$y(0) = 5 \Rightarrow A = 5$, so $y = 5e^{-x}$.

12. **Correct answer: (D) $x^x + C$** **1**

$d/dx(x^x) = x^x(1 + \log x)$ (by logarithmic differentiation).

So $\int x^x(1 + \log x)\,dx = x^x + C$.

13. **Correct answer: (B) 1** **1**

$\hat{i} \times \hat{j} = \hat{k}$, so $(\hat{i} \times \hat{j}) \cdot \hat{k} = \hat{k} \cdot \hat{k} = 1$.

$\hat{i} \cdot \hat{j} = 0$; the value is 1 + 0 = 1.

14. **Correct answer: (C) $\frac{\pi}{3}$** **1**

$\tan\theta = \frac{\square}{\vec{a} \times \vec{b}} = \sqrt{3}$.

$\theta = \frac{\pi}{3}$.

15. **Correct answer: (A) $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$** **1**

A line through $\vec{a}$ parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

Here $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

16. **Correct answer: (D) independent** **1**

$P(A)\,P(B) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$.

This equals $P(A \cap B)$, so A and B are independent.

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17. **Correct answer: (C) $\frac{1}{3}$** **1**

Given odd: {1, 3, 5}; among these only 5 is greater than 4.

Required probability = $\frac{1}{3}$.

18. **Correct answer: (A) 2** **1**

Mean = $\Sigma xP(x) = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.3 + 3 \times 0.4$.

$= 0.2 + 0.6 + 1.2 = 2$.

19. **Correct answer: (A)** **1**

R is a standard property of definite integrals — R is true.

By R, the integral = $\int_0^1 (1-x)\,x^{99}\,dx = \frac{1}{100} - \frac{1}{101} = \frac{1}{10100}$. A is true and R explains it.

20. **Correct answer: (A)** **1**

R is correct: dividing by y and multiplying by dx separates the variables — R is true.

Integrating, $\log y = \log x + \log c \Rightarrow y = cx$. A is true and R explains it.

SECTION B

21. **Solution:**

$\sin^6x + \cos^6x = (\sin^2x + \cos^2x)^3 - 3\sin^2x\cos^2x(\sin^2x + \cos^2x) = 1 - 3\sin^2x\cos^2x$; so the integrand = $\sec^2x + \operatorname{cosec}^2x - 3$. **1**

$\int = \tan x - \cot x - 3x + C$. **1**

$\therefore$ **$\tan x - \cot x - 3x + C$**

OR

e^(|x|) is an even function, so the integral = $2\int_0^1 e^x\,dx$. **1**

$= 2[e^x]$ from 0 to 1 $= 2(e - 1)$. **1**

$\therefore$ **$2(e - 1)$**

22. **Solution:**

$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}} \times \left(1 + \frac{x}{\sqrt{x^2+a^2}}\right)$. **1**

$= \frac{1}{x+\sqrt{x^2+a^2}} \times \frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$. **1**

$\therefore$ **$1/\sqrt{(x^2 + a^2)}$**

23. **Solution:**

(A) Three Dimensional Geometry

$a_1a_2 + b_1b_2 + c_1c_2 = (\sqrt{3} - 1) + (-\sqrt{3} - 1) + 8 = 6$; magnitudes $\sqrt{6}$ and $\sqrt{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2+16} = \sqrt{24}$. **1**

$\cos\theta = \frac{6}{\sqrt{6}\times\sqrt{24}} = \frac{6}{12} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$. **1**

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∴ **π/3**

OR

(B) Vector Algebra

(i) Parallel: $\frac{3}{1} = \frac{2}{p} = \frac{9}{3} \Rightarrow p = \frac{2}{3}$. **1**

(ii) Perpendicular: $3 + 2p + 27 = 0 \Rightarrow p = -15$. **1**

∴ **(i) 2/3 (ii) −15**

24. Solution:

$\overrightarrow{AB} = 4\hat{i} + 2\hat{j} + 6\hat{k}$ and $\overrightarrow{BC} = 6\hat{i} + 3\hat{j} + 9\hat{k} = \frac{3}{2}\overrightarrow{AB}$; they are parallel with common point B, so A, B, C are collinear. **1**

$|\overrightarrow{AB}| : |\overrightarrow{BC}| = 1 : \frac{3}{2} = 2 : 3$, so B divides AC internally in the ratio 2 : 3. **1**

∴ **Collinear; 2 : 3**

25. Solution:

Corner points of the feasible region: (1, 5), (4, 2), (4, 5). **1**

Z = 40, 34, 55 respectively; minimum Z = 34 at (4, 2). **1**

∴ **Min Z = 34 at (4, 2)**

SECTION C

26. Solution:

$|A| = 1(0 - 2) - 1(1 - 6) + 1(1 - 0) = -2 + 5 + 1 = 4 \neq 0$, so A^{-1} exists. **1**

Cofactors: $A_{11} = -2$, $A_{12} = 5$, $A_{13} = 1$; $A_{21} = 0$, $A_{22} = -2$, $A_{23} = 2$; $A_{31} = 2$, $A_{32} = -1$, $A_{33} = -1$; **1**

$$\text{adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}.$$

$$A^{-1} = (1/|A|)\text{ adj } A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}.$$ **1**

∴ **A^{-1} = (1/4)[[−2, 0, 2], [5, −2, −1], [1, 2, −1]]**

27. Solution:

Let x m be used for the square and (28 − x) m for the circle. $A = \left(\frac{x}{4}\right)^2 + \frac{(28 - x)^2}{4\pi}$. **1**

$\frac{dA}{dx} = \frac{x}{8} - \frac{28 - x}{2\pi} = 0 \Rightarrow \pi x = 4(28 - x) \Rightarrow x = \frac{112}{\pi + 4}$; $\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{1}{2\pi} > 0$, so A is minimum. **1**

Square: $\frac{112}{\pi + 4}$ m; circle: $28 - \frac{112}{\pi + 4} = \frac{28\pi}{\pi + 4}$ m. **1**

∴ **112/(π + 4) m and 28π/(π + 4) m**

28. Solution:

Let $u = (\sin x)^x$: $\log u = x \log \sin x \Rightarrow du/dx = (\sin x)^x(x \cot x + \log \sin x)$. **1**

Let $v = \sin^{-1}\sqrt{x}$: $\frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}\sqrt{1-x}}$. **1**

$\frac{dy}{dx} = (\sin x)^x(x \cot x + \log \sin x) + \frac{1}{2\sqrt{x-x^2}}$. **1**

∴ **$(\sin x)^x(x \cot x + \log \sin x) + 1/(2\sqrt{(x - x^2)})$**

OR

$m \log x + n \log y = (m + n) \log(x + y)$. **1**

$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\left(1+\frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx}\left(\frac{n}{y} - \frac{m+n}{x+y}\right) = \frac{m+n}{x+y} - \frac{m}{x}$. **1**

$\frac{dy}{dx}\frac{nx-my}{y(x+y)} = \frac{nx-my}{x(x+y)} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$. Hence proved. **1**

∴ **Proved**

29. Solution:

$\frac{dx}{dy} = x + y \Rightarrow \frac{dx}{dy} - x = y$, linear in x; IF $= e^{-y}$. **1**

$x e^{-y} = \int y e^{-y}\, dy = -y e^{-y} - e^{-y} + C$ (by parts). **1**

$x = -(y + 1) + C e^y$, i.e. $x + y + 1 = C e^y$. **1**

∴ **$x + y + 1 = Ce^y$**

OR

$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2} = v + \frac{v^2}{2}$ with $y = vx$; $v + x\frac{dv}{dx} = v + \frac{v^2}{2} \Rightarrow \frac{2\,dv}{v^2} = \frac{dx}{x}$. **1**

$-\frac{2}{v} = \log|x| + C \Rightarrow -\frac{2x}{y} = \log|x| + C$. **1**

$x = 1, y = 2 \Rightarrow C = -1$; $y = \frac{2x}{1 - \log|x|}$. **1**

∴ **$y = 2x/(1 - \log|x|)$**

30. Solution:

Direction of BC: $2, -2, -4 \propto 1, -1, -2$; any point of BC: $P(t, -1 - t, 3 - 2t)$. **1**

AP $= (t - 1, -9 - t, -1 - 2t)$; AP ⊥ BC: $(t - 1) + (9 + t) + (2 + 4t) = 6t + 10 = 0 \Rightarrow t = -5/3$. **1**

Foot of the perpendicular: $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$. **1**

∴ **(−5/3, 2/3, 19/3)**

OR

Take a cube of side a with one vertex at the origin; its diagonals have direction ratios (1, 1, 1), (−1, 1, 1), (1, −1, 1), (1, 1, −1). **1**

If l, m, n are the direction cosines of the line, $\cos\alpha = (l + m + n)/\sqrt{3}$, $\cos\beta = (-l + m + n)/\sqrt{3}$, $\cos\gamma = (l - m + n)/\sqrt{3}$, $\cos\delta = (l + m - n)/\sqrt{3}$. **1**

Sum of squares $= (1/3)\cdot 4(l^2 + m^2 + n^2) = 4/3$ (cross terms cancel and $l^2 + m^2 + n^2 = 1$). Hence proved. **1**

∴ **Proved**

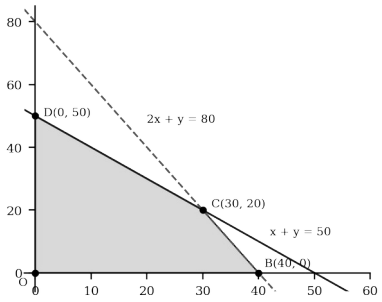
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31. Solution:

Draw x + y = 50 and 2x + y = 80; they meet at C(30, 20). The feasible region is OBCD with O(0, 0), B(40, 0), C(30, 20), D(0, 50). **1**

Z at the corners: O: 0; B: 4,20,000; C: 4,95,000; D: 4,50,000. **1**

Maximum Z = 4,95,000 at x = 30, y = 20. **1**



∴ **Max Z = 4,95,000 at (30, 20)**

SECTION D

32. Solution:

Reflexive: f(a) = f(a) for every a ∈ X, so (a, a) ∈ R. **1**

Symmetric: (a, b) ∈ R ⇒ f(a) = f(b) ⇒ f(b) = f(a) ⇒ (b, a) ∈ R. **1**

Transitive: f(a) = f(b) and f(b) = f(c) ⇒ f(a) = f(c); so R is an equivalence relation. **1**

For $f(x) = x^2$, b is related to 3 iff $b^2 = 3^2 = 9$. **1**

So the set of elements related to 3 is {−3, 3}. **1**

∴ **Equivalence relation; {−3, 3}**

OR

(i) $f(x_1) = f(x_2) \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$, so f is one-one. **1**

f(x) = 2x is always even; an odd natural number such as 1 has no pre-image, so f is not onto. **1**

(ii) g(1) = g(2) = 1 with 1 ≠ 2, so g is not one-one. **1**

For any m ∈ N, g(m + 1) = m (since m + 1 ≥ 2). **1**

So every element of N has a pre-image and g is onto. **1**

∴ **Shown**

33. Solution:

Let $I = \int_0^{\pi} x \sin^3 x\,dx$. Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$: $I = \int_0^{\pi} (\pi - x)\sin^3 x\,dx$. **1**

Adding: $2I = \pi \int_0^{\pi} \sin^3 x\,dx \Rightarrow I = \frac{\pi}{2}\int_0^{\pi} \sin^3 x\,dx$. **1**

$\sin^3 x = \frac{3\sin x - \sin 3x}{4}$, so $\int_0^{\pi} \sin^3 x\,dx = \frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right]$ from 0 to π. **1**

$= \frac{1}{4}\left[\left(3 - \frac{1}{3}\right) - \left(-3 + \frac{1}{3}\right)\right] = \frac{1}{4} \times \frac{16}{3} = \frac{4}{3}$. **1**

$I = \frac{\pi}{2} \times \frac{4}{3} = \frac{2\pi}{3}$. **1**

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$\therefore$ **$2\pi/3$**

34. Solution:

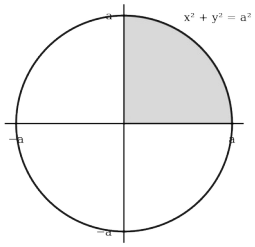
The circle is symmetric about both axes, so Area = 4 × (area in the first quadrant), where $y = \sqrt{a^2 - x^2}$ (see figure). **1**

Area $= 4\int_0^a \sqrt{a^2 - x^2}\,dx$. **1**

$= 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$ from 0 to a. **1**

$= 4\left[0 + \frac{a^2}{2} \times \frac{\pi}{2} - 0\right]$. **1**

$= \pi a^2$ square units. **1**



$\therefore$ **πa^2 sq. units**

OR

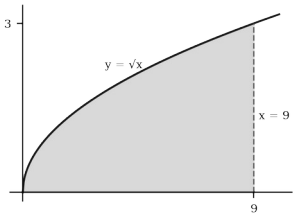
$y = \sqrt{x} \geq 0$; the region lies above the x-axis between x = 0 and x = 9 (see figure). **1**

Area $= \int_0^9 \sqrt{x}\,dx$. **1**

$= \left[\frac{2}{3}x^{(3/2)}\right]$ from 0 to 9. **1**

$= \frac{2}{3} \times 27$. **1**

= 18 square units. **1**



$\therefore$ **18 sq. units**

35. Solution:

X can take the values 0, 1 and 2. **1**

(a) $P(X = 0) = \frac{7}{10} \times \frac{6}{9} = \frac{7}{15}$; $P(X = 1) = \frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{3}{9} = \frac{7}{15}$. **1**

$P(X = 2) = \frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$ (total = 1). **1**

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(b) Mean = $0 \times \frac{7}{15} + 1 \times \frac{7}{15} + 2 \times \frac{1}{15} = \frac{9}{15} = \frac{3}{5}$. **1**

(c) $P(X \geq 1) = 1 - \frac{7}{15} = \frac{8}{15}$. **1**

∴ **(a) 7/15, 7/15, 1/15 (b) 3/5 (c) 8/15**

SECTION E

36. Solution:

(i)

OG = $\sqrt{36+9+4}$ = 7 m. **1**

∴ **7 m**

(ii)

Direction cosines $\frac{6}{7}, \frac{3}{7}, \frac{2}{7}$. **1**

(iii)(a)

Direction ratios of AE: 0 − 6, 3 − 0, 2 − 0 = −6, 3, 2 (magnitude 7); $\cos\theta = \frac{|6(-6)+3(3)+2(2)|}{7 \times 7} = \frac{23}{49}$. **1**

$\theta = \cos^{-1}\left(\frac{23}{49}\right)$. **1**

∴ **$\cos^{-1}(23/49)$**

OR

(iii)(b)

OG: (6t, 3t, 2t); AE: (6 − 6s, 3s, 2s). From y and z: t = s; from x: 6t = 6 − 6t ⇒ t = 1/2. **1**

All three coordinates agree, so the strings intersect at (3, 3/2, 1), the centre of the hall. **1**

∴ **(3, 3/2, 1)**

37. Solution:

(i)

$h'(x) = \frac{1}{3}(3x^2 - 18x + 24) = x^2 - 6x + 8$. **1**

(ii)

h′(x) = (x − 2)(x − 4) = 0 ⇒ x = 2 and x = 4. **1**

∴ **x = 2, 4**

(iii)(a)

h″(x) = 2x − 6: h″(2) = −2 < 0 (local maximum), h″(4) = 2 > 0 (local minimum). **1**

Local maximum height $h(2) = \frac{20}{3}$ m; local minimum height $h(4) = \frac{16}{3}$ m. **1**

∴ **20/3 m and 16/3 m**

OR

(iii)(b)

h′(x) = (x − 2)(x − 4) > 0 for 0 < x < 2 and 4 < x < 6; h′(x) < 0 for 2 < x < 4. **1**

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The track rises on (0, 2) and (4, 6), and falls on (2, 4). **1**

∴ **Rising (0, 2) ∪ (4, 6); falling (2, 4)**

38. Solution:

(i)

$2x + 3y + z = 200$, $x + 2y + 3z = 260$, $3x + y + 2z = 260$; $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 200 \\ 260 \\ 260 \end{bmatrix}$. **1**

$|A| = 2(4 - 3) - 3(2 - 9) + 1(1 - 6) = 2 + 21 - 5 = 18$. **1**

∴ **$|A| = 18$**

(ii)

$X = A^{-1}B = (1/|A|)(\text{adj } A)B = \frac{1}{18}\begin{bmatrix} 200 - 1300 + 1820 \\ 1400 + 260 - 1300 \\ -1000 + 1820 + 260 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 720 \\ 360 \\ 1080 \end{bmatrix}$. **1**

$x = 40$, $y = 20$, $z = 60$: rose ₹40, marigold ₹20, tulip ₹60. **1**

∴ **₹40, ₹20, ₹60**

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